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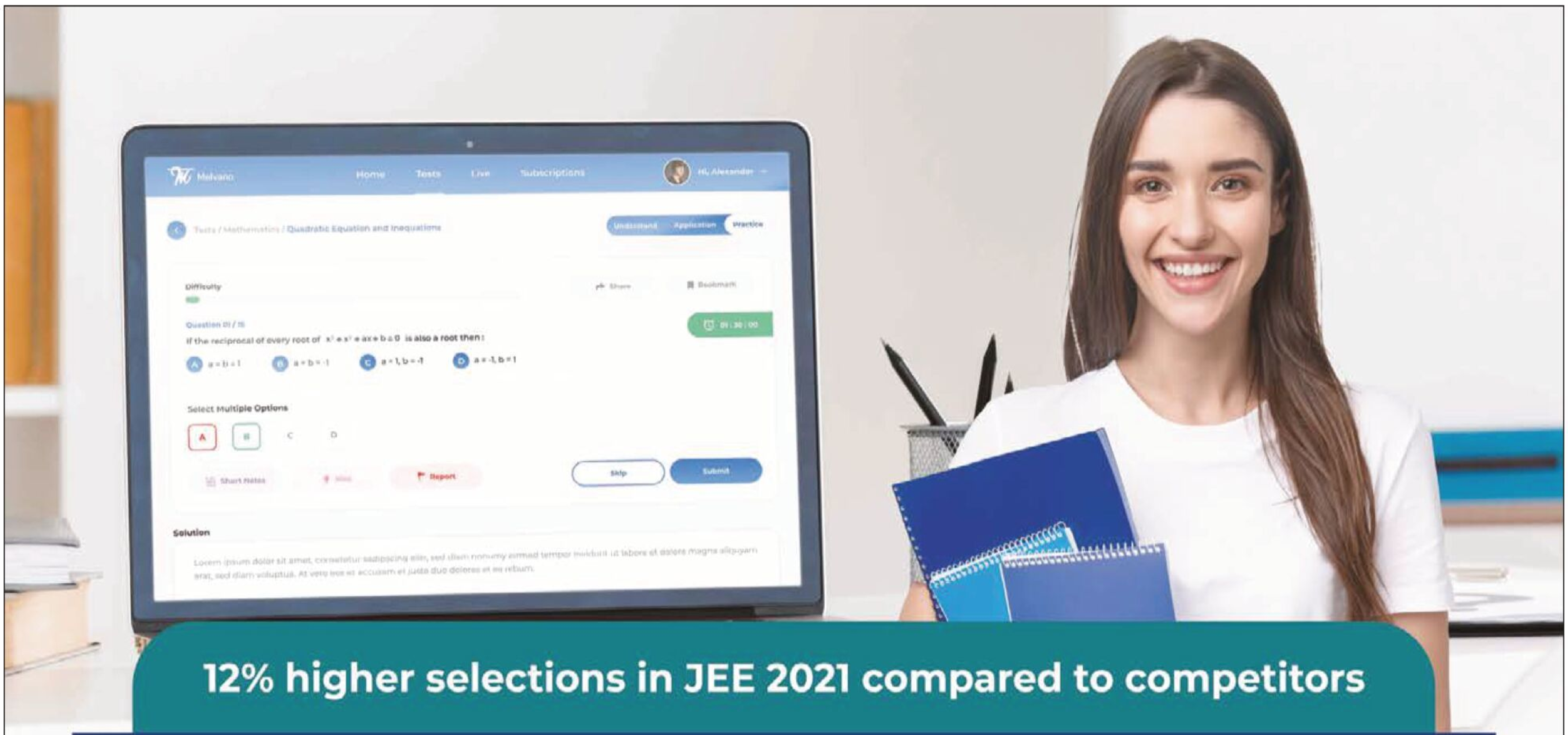

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The logical side of Physics

Amazon Academy's Physics teacher Ashish Gupta talks about his perspectives on how to tackle Physics for JEE & NEET syllabus.

Ashish Gupta had two passions in his life: Physics and teaching. He made the most of both and decided to become a teacher. Nine years on, Ashish Sir, as he is popularly known to his students, is happy he decided to act on his first two loves. As Amazon Academy's Physics faculty, Ashish Sir explains how Physics is the most practical subject among the all JEE subjects. He says Physics needs a fair amount of logical understanding of concepts and significant practice. Application is as important as theory, he emphasizes in this informative tête-à-tête.

Take us through your educational years Ashish Sir!

Sure. I did my schooling till 10th from Jodhpur, Rajasthan. My favorite subject kept fluctuating between Physics and Maths till 10th standard. I went to Kota to prepare for JEE for a couple of years. In 11th standard, I understood that while Maths was a comprehensive subject, Physics made me happy. It had a good balance of logic, understanding and numerical. In 12th, I cleared the JEE in my first attempt and went to IIT Bombay to pursue B.Tech. in Mechanical Engineering before graduating in 2009.

What did you do after that?

After my graduation, I got placed in a company where we used to trade derivatives. I did that for a couple of years but I was always passionate about teaching. Physics always excited me as a subject. So, I decided to act upon my passion and in 2012, joined FIITJEE as a teacher. I also taught at IITian's Pace in Nagpur for three years. Then I was in Indore for the last three years at Allen Career Institute, before joining Amazon Academy. I have been a teacher for the last nine years now.

Tell us what are the topics a student should study in JEE & NEET?

I would say that Optics and Modern Physics are the topics which are important. They are concise in nature; they might not be exhaustive but the number of questions that come from those topics are quite high. Apart from that, Electromagnetic Induction, Alternating Current, and Magnetism also contribute significantly to the number of questions that you see in the papers. All of these are Class 12th topics. For 11th, Thermodynamics is a very good topic — it's a concise topic but then again, students tend to neglect it. The number of questions that come from Thermodynamics is quite high.

For JEE Main, the same topics are high priority. I would also add Semiconductors, Communication, Polarization, Diffraction – topics that appear only in Main. JEE tends to ask questions from these topics every year and the level of questions is also not that high. So, if students devote just even a day in learning these topics, they tend to do well. In turn, this helps them score better.

Which books should the students refer to?

There are certain standard books that everybody studies from. One of them is Concept of Physics by H.C. Verma. It's a good book for basic understanding. For basic concepts, I would recommend a book by Resnick, Halliday, and Walker i.e. Principles of Physics. This is again a book which focuses on the concepts.

As for problem solving and practice purposes, I would say H.C. Verma is a good book. Apart from that, we have our own Amazon Academy platform that deals with questions on each topic as well as part and full tests that are available for free. I can vouch that the level of these questions is amazingly good. One more source which is usually underrated but very reliable is the previous years' questions. So, the last 40-45 years of question banks should be accessed. This helps the students check their understanding of topics. It's an absolute must.

What about revision? Any tips on that?

You must remember that not everyone who practices gets good marks. I cannot emphasize enough on revisiting and revising previous topics. Let me explain: Suppose you have learnt a topic today and you do not go through it for the next two months, there is a good chance that you will forget it. My recommendation is to solve at least 15 to 20 questions after two-three weeks. This will help you dust up on what you studied earlier. So, structure your academics in a way that you can engage in questions of the previous topics along with the current topics.

Anything else you would like to add?

See, there is a saying in Physics, 'Logic is Magic'. If you are a logical person, then you have a good chance of cracking Physics as a subject. But I also feel that we overemphasize on logic sometimes. Students feel that they must grasp a concept first, understand it, then implement it. It's not like that. I think the majority of students miss the trick by focusing too much on concepts and not working hard enough on their skill set. If you practice a lot in Physics, you will fall in love with the subject. You will realize that the questions are so practical. So be mentally ready to practice… practice hard.

Disclaimer: The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

◆ ◆

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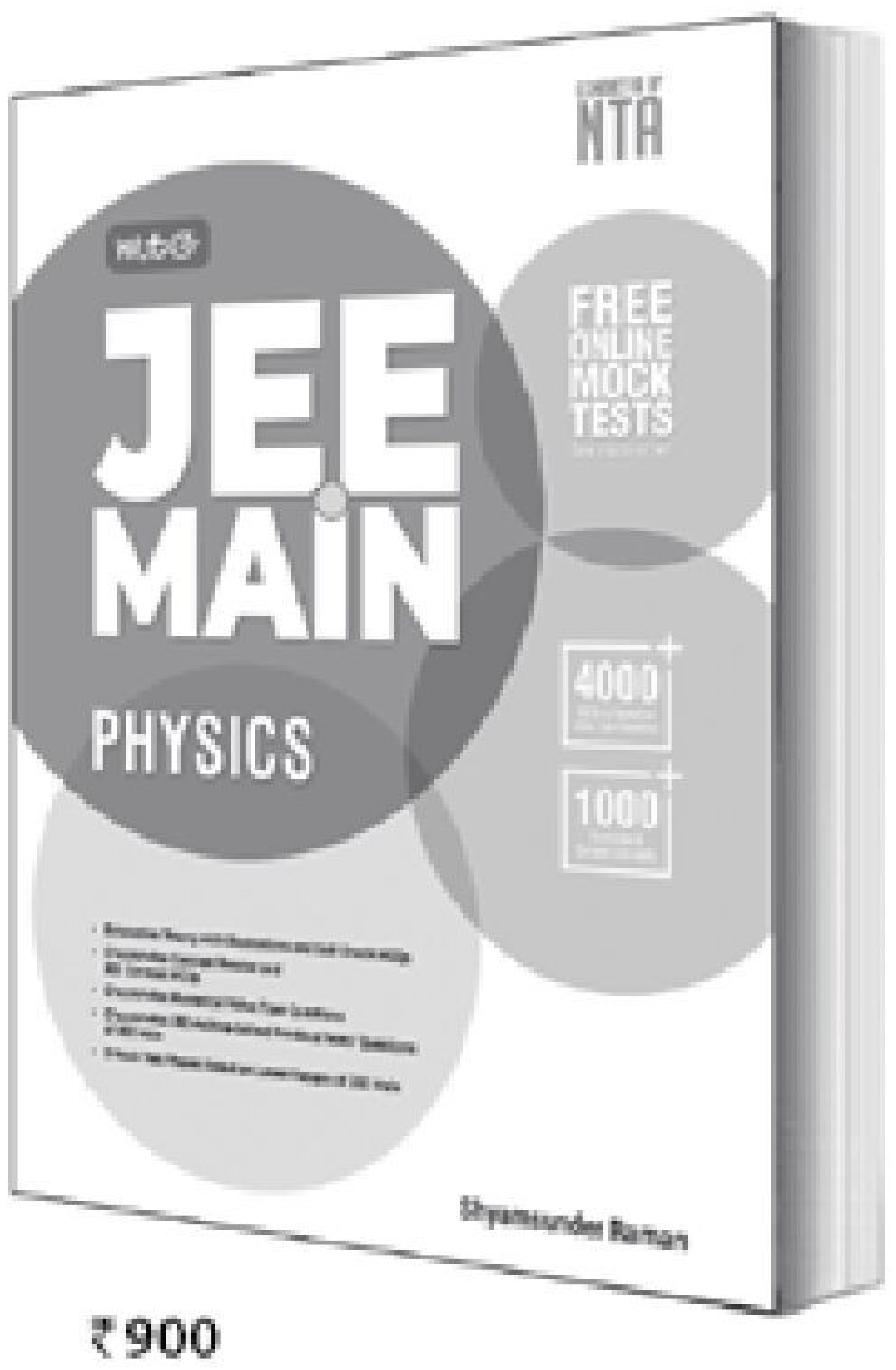
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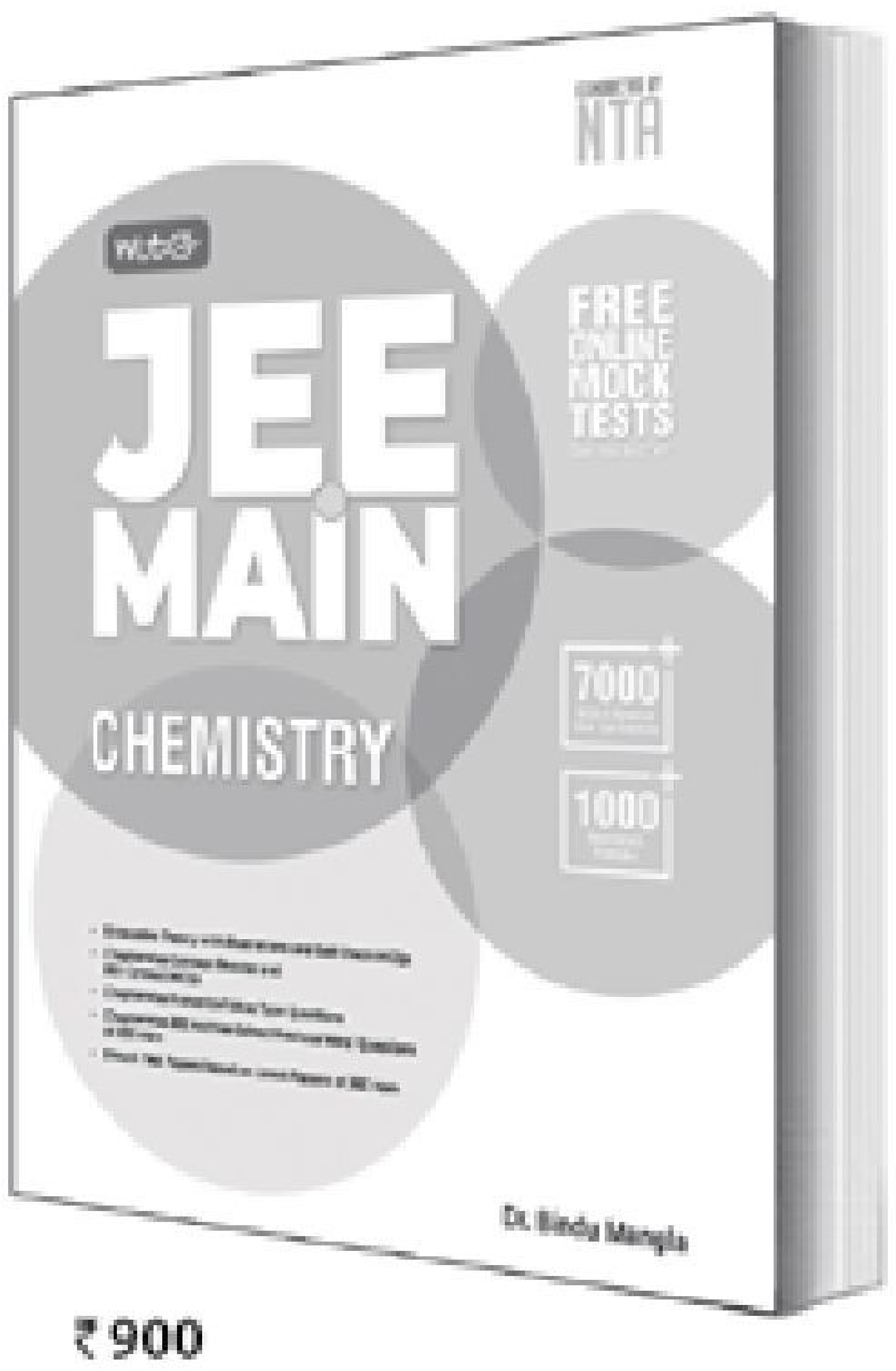
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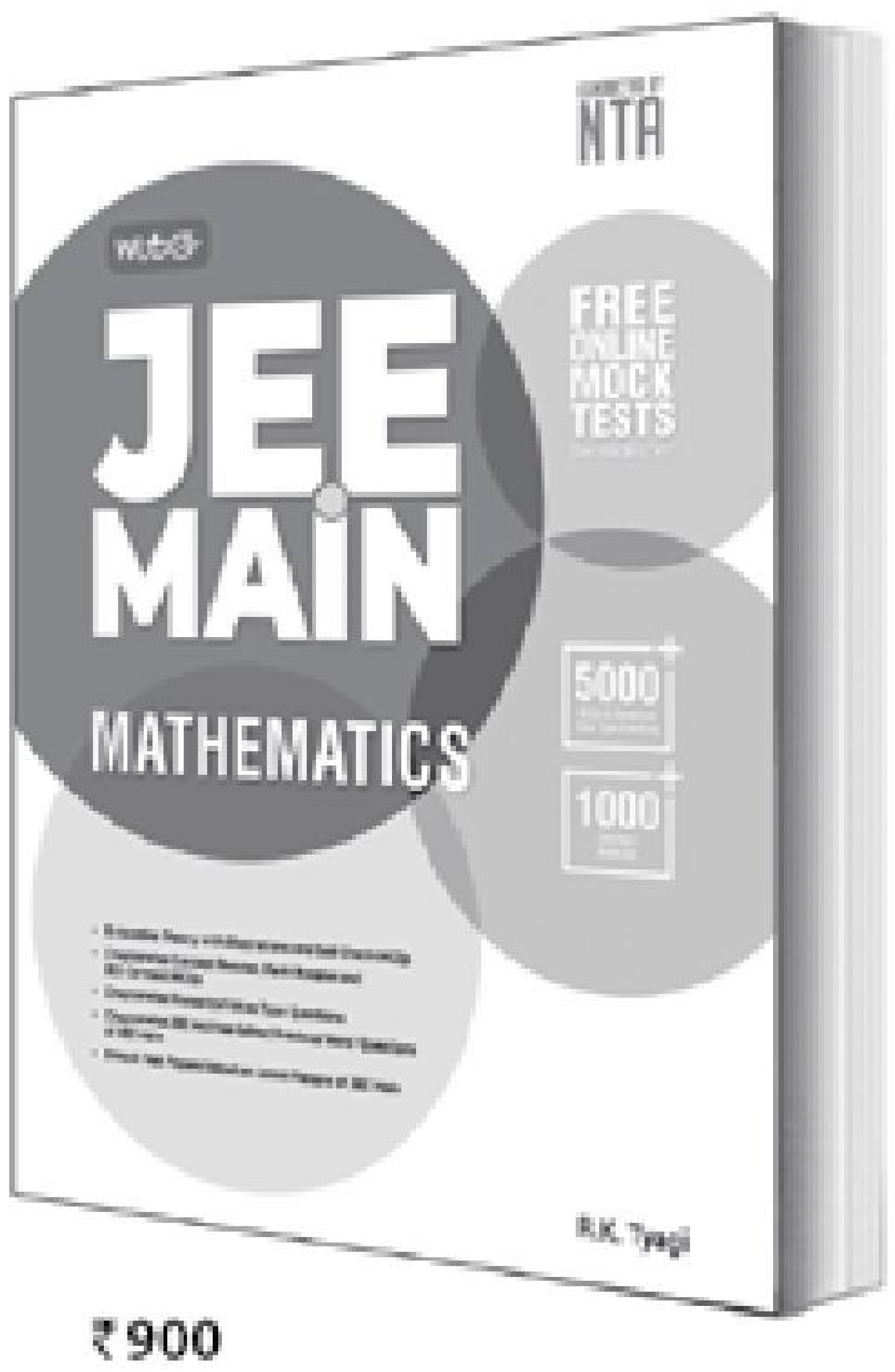
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CONTENTS

Competition Edge

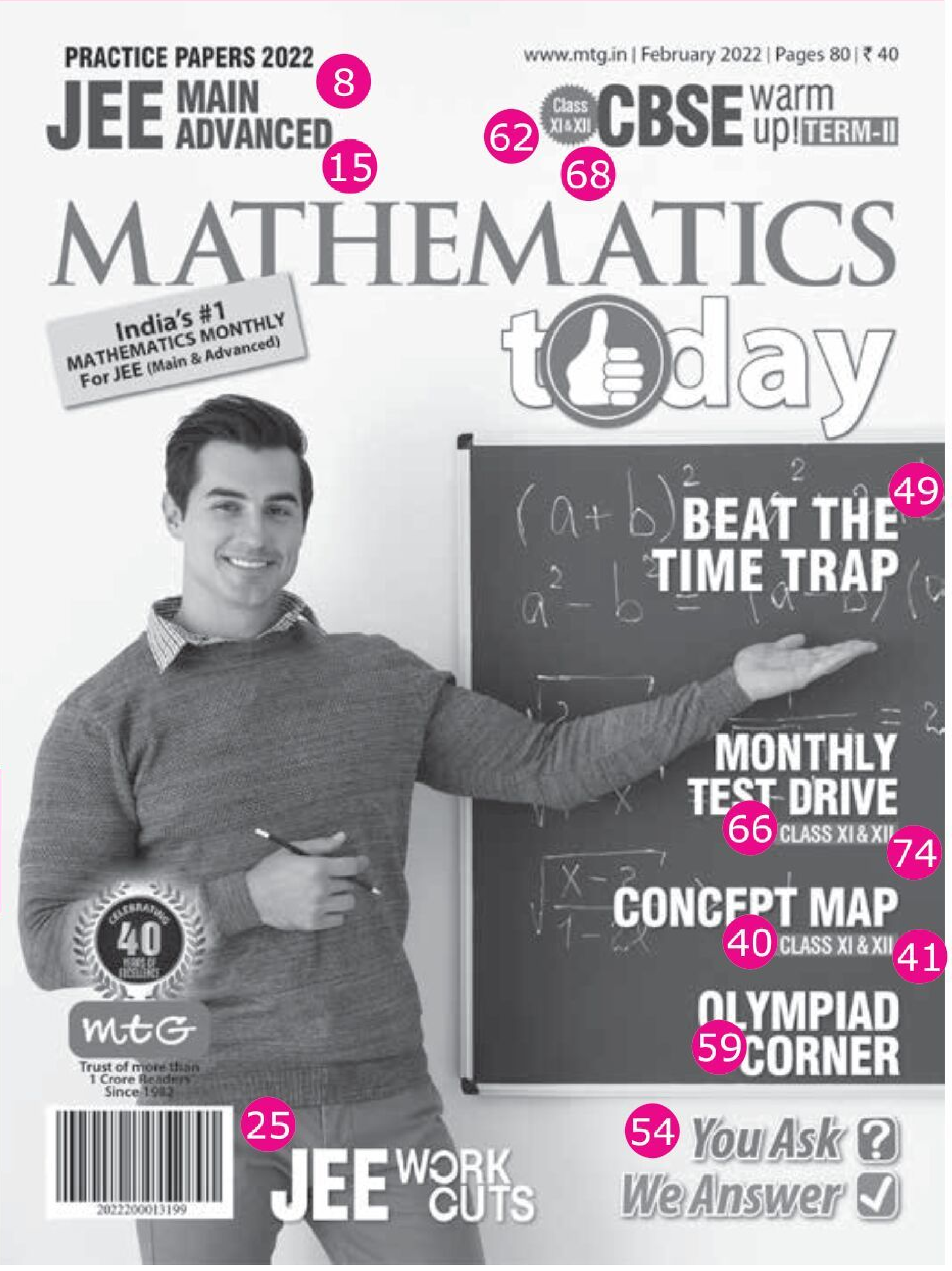
8 JEE Main
Practice Paper
15 JEE Advanced
Practice Paper
25 JEE Work Outs
34 Brush Up for JEE
49 Beat The Time Trap
54 You Ask We Answer
55 Challenging Problems
59 Olympiad Corner

Class XI

40 Concept Map
Circles
62 CBSE Warm Up!
Conic Section | Introduction to Three Dimensional Geometry | Derivatives | Probability
66 Monthly Test Drive
Straight Lines

Class XII

41 Concept Map
Inverse Trigonometric Functions
68 CBSE Warm Up!
Vector | Three Dimensional Geometry | Probability
74 Monthly Test Drive
Differential Equations



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PRACTICE PAPER 2022

JEE MAIN

SINGLE OPTION CORRECT TYPE

1. If $xy = e - e^y$, then $\left[\frac{d^2y}{dx^2}\right]_{x=0}$ is equal to

(a) $\frac{1}{e}$ (b) $\frac{1}{e^3}$

(c) $\frac{1}{e^2}$ (d) None of these

2. If $f(x)=\begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$, then

(a) $f'(x) = 1$

(b) $y = f(x)$ is a straight line parallel to y-axis

(c) $\int_0^2 f(x)\,dx = 8a^6$ (d) None of these

3. If a, b, c are real numbers and ω is a complex cube root of unity such that $a + b\omega + c\omega^2 = 0$, then

(a) $b = -c$ (b) $a = 0, b = c$

(c) $a = 1, b = c$ (d) $a = b = c$

4. Five digit numbers divisible by 3 are formed using the digits 0, 1, 2, 3, 4, 6, 7 without repetition. The number of such numbers is

(a) 936 (b) 480 (c) 600 (d) 216

5. In a ΔABC, if $s = 3+\sqrt{3}+\sqrt{2}$, $3B - C = 30°$ $A + 2B = 120°$, then the largest side is

(a) 2 (b) $2\sqrt{2}$

(c) $2(\sqrt{3}+1)$ (d) $\sqrt{3}-1$

6. A letter is taken out at random from 'ASSISTANT' and another letter taken out from the letters of the word 'STATISTICS'. The probability that they are identical letters, is

(a) $\frac{13}{90}$ (b) $\frac{1}{45}$

(c) $\frac{19}{90}$ (d) None of these

7. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $(A(\text{adj } A)A^{-1})A =$

(a) $2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1/6 & -1/6 \\ 2/6 & 1/6 & 3/6 \\ 3/6 & 2/6 & 1/6 \end{bmatrix}$ (d) None of these

8. If X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y|X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$, then which of the following is incorrect?

(a) $P(X \cup Y) = 2/3$

(b) X and Y are independent

(c) $P(X^c \cap Y) = 1/6$

(d) X and Y are not independent

9. Let $\vec{a} = \alpha_1\hat{i} + \alpha_2\hat{j} + \alpha_3\hat{k}$, $\vec{b} = \beta_1\hat{i} + \beta_2\hat{j} + \beta_3\hat{k}$ and $\vec{c} = \gamma_1\hat{i} + \gamma_2\hat{j} + \gamma_3\hat{k}$, $|\vec{a}| = 2\sqrt{2}$, $\vec{a}$ makes angle $\frac{\pi}{3}$ with plane of $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}^n$ is equal to (n is even natural number)

(a) $\left(\frac{|\vec{b}||\vec{a}|}{6}\right)^{n/2}$ (b) $\left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$

(c) $\frac{\left(|\vec{b}||\vec{c}|\right)^{n/2}}{\sqrt{32^n}}$ (d) None of these

10. If α, β are roots of the equation $p(x^2 - x) + x + 5 = 0$ and p_1, p_2 are two values of p for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then the value of $\frac{p_1}{p_2} + \frac{p_2}{p_1} =$

(a) 254 (b) 0 (c) 245 (d) −254

11. If the sum of the coefficients in the expansion of $(b+c)^{20}[1+(a-2)x]^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2bc\,x-(b+c)y]^{10}$, where a, b, c are positive constants, then

(a) $a \geq \sqrt{bc}$ (b) $a=\dfrac{2bc}{b+c}$

(c) $a^2=cb$ (d) $\dfrac{1}{c},\dfrac{1}{a},\dfrac{1}{b}$ are in G.P.

12. If $(x^2+y^2)dy=xydx$, $y(1)=1$, and $y(x_0)=e$, then $x_0=$

(a) $\sqrt{2(e^2-1)}$ (b) $\sqrt{2(e^2+1)}$

(c) $\sqrt{3}e$ (d) $\sqrt{\dfrac{e^2+1}{2}}$

13. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is

(a) 0.534 (b) 0.466

(c) 0.763 (d) None of these

14. Let $A=\{x\in R: -1\leq x\leq 1\}=B$. Then the mapping $f: A\to B$ given by $f(x)=x|x|$ is

(a) injective but not surjective

(b) surjective but not injective

(c) bijective

(d) none of these

15. The length of the common chord of the circles $x^2+y^2-2x+4y-20=0$, $x^2+y^2+6x+10y-2=0$ is

(a) 8 (b) 10 (c) $\dfrac{48}{5}$ (d) $\dfrac{24}{5}$

16. A bag contains 12 pairs of socks, 4 socks are picked up at random. Then, the probability that there is at least one pair, is

(a) $\dfrac{41}{161}$ (b) $\dfrac{120}{161}$

(c) $\dfrac{21}{161}$ (d) None of these

17. Let $x>0$, $y>0$, $z>0$ are respectively the 2nd, 3rd, 4th terms of a G.P and

$$\Delta=\begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2}\end{vmatrix}=(r-1)^2\left(1-\frac{1}{r^2}\right)$$

(where r is the common ratio), then

(a) $k=-1$ (b) $k=1$

(c) $k=0$ (d) None of these

18. If $x=\sin t-\operatorname{cosec} t$, $y=\sin^5 t-\operatorname{cosec}^5 t$, then $(x^2+4)\,y''=$

(a) $xy'-5y$ (b) $-xy'+5y$

(c) $xy'-25y$ (d) $-xy'+25y$

19. If $\dfrac{\pi a}{2}+2(a+b)=L$, a constant, then the function $\dfrac{\pi a^2}{8}+3ab$ is maximum when $\dfrac{b}{a}=$

(a) $1+\dfrac{\pi}{3}$ (b) $1-\dfrac{\pi}{3}$ (c) $1+\dfrac{\pi}{6}$ (d) $1-\dfrac{\pi}{6}$

20. If $\dfrac{dy}{dx}+y\cos x=\dfrac{1}{2}\sin 2x$, $y(0)=1$, then $y\left(\dfrac{\pi}{2}\right)=$

(a) 1 (b) e (c) $\dfrac{1}{e}$ (d) $\dfrac{2}{e}$

NUMERICAL ANSWER TYPE

21. Each of two boxes contains both black and white marbles, and the total number of marbles in the two boxes is 25. One marbles is taken out of each box. The probability that both marbles are black is $\dfrac{27}{50}$. If the probability that both marbles are white is $\dfrac{m}{n}$, where m, n, are relatively prime positive integers, then m is ________.

22. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 2 and 3 respectively satisfy the relation $|[\vec{a}\ \vec{b}\ \vec{c}]|=6$. Then the value of $|(\vec{a}\times\vec{c})\cdot\vec{d}|^2+|(\vec{a}\times\vec{c})\times\vec{d}|^2$, where $\vec{d}$ is unit vector, is equal to ________.

23. A circle of radius R touches externally a set of 12 circles each of radius r surrounding it. Each one of the smaller circles touches two circles of the set. Then $\dfrac{R}{r}=\sqrt{m}+\sqrt{n}-1$, where $m, n\in N$ and $m+n$ is ________.

24. If $y=\sin(2\sin^{-1}x)$, then the value of $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+4y$ is ________.

25. The area (in sq. units) of the quadrilateral formed by the tangents at the end-points of latus rectum of the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1$ is ________.

26. A spherical balloon is being inflated at the rate of 35cm^3/min. Then, the rate of increase of surface area (in cm^2 /min) of the balloon when its diameter is 14 cm is equal to ________.

27. The number of integral values of k for which the equation $3\cos x + 4\sin x = 2k + 1$ has a solution, is ________.

28. If A is the area of the greater region bounded by the curve $y = \cos x$, $y = x + 1$ and $y = 0$ in sq. units, then value of $6A$ is ________.

29. If $\int \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx = A\sin^{-1}x + Bx\sqrt{1-x^2}$ $+ C$, then $A + B =$ ________.

30. Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$. An Element (a,b) of their cartesian product $A \times B$ is chosen at random. If the probability that $\{a + b = 9\}$ is $\frac{p}{q}$, then $\frac{q}{p}$ is equal to ________.

SOLUTIONS

1. **(c) :** We have, $xy = e - e^y$ ...(i)

$$\Rightarrow\quad y + x\frac{dy}{dx} = -e^y\frac{dy}{dx} \Rightarrow e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0 \qquad \text{...(ii)}$$

$$\therefore\quad \left[\frac{dy}{dx}\right]_{x=0} = -\frac{1}{e}\ [\text{As } x = 0 \Rightarrow y = 1 \text{ by(i)}]$$

Again differentiating (ii) w.r.t. x, we get

$$e^y\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0 \qquad \text{...(iii)}$$

Putting $x = 0$, $y = 1$ and $\frac{dy}{dx} = -\frac{1}{e}$ in (iii), we get

$$\frac{d^2y}{dx^2} = \frac{1}{e^2}$$

2. **(c) :** We have,

$$f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1x$ and $C_3 \to C_3 - C_2x$

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 3x & 2a^2 & 4a^2x \\ 3x^2+2a^2 & 4a^2x & 6a^2x^2+2a^4 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1x$ and $R_3 \to R_3 - R_2x$

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2a^2 & 2a^2x \\ 2a^2 & 2a^2x & 2a^2x^2+2a^4 \end{vmatrix} = a^4\begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2 & 2x \\ 2 & 2x & 2x^2+2a^2 \end{vmatrix}$$

$$= a^4[3(4x^2 + 4a^2 - 4x^2) + 2a^2(-4)] = a^4(12a^2 - 8a^2) = 4a^6$$

$$\therefore\ f(x) = 4a^6 \Rightarrow f'(x) = 0 \text{ and } \int_0^2 4a^6dx = 4a^6[x]_0^2 = 8a^6$$

3. **(d):** $a + b\,\omega + c\,\omega^2 = 0$

$$\Rightarrow\ a + b\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) + c\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = 0$$

Equating real and imaginary parts, we get

$a = \frac{b+c}{2}$, $b = c$.

$\therefore\quad a = b = c$

4. **(c) :** The possible ways of choosing the 5 digits are (1, 3, 4, 6, 7), (0, 1, 2, 3, 6), (0, 1, 3, 4, 7), (0, 1, 4, 6, 7), (0, 2, 3, 6, 7), (0, 2, 3, 4, 6)

$\therefore$ Required number of numbers

$= 5! + 5\,(5! - 4!) = 120 + 5 \times 96 = 600$

5. **(c) :** $3B - C = 30°$ ...(i) $\quad A + 2B = 120°$ ...(ii)

$A + B + C = 180°$ ...(iii)

Solving (i), (ii) & (iii), we get $A = 30°$, $B = 45°$, $C = 105°$

$$\text{Now, } a = \frac{k}{2}, b = \frac{k}{\sqrt{2}}, c = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k$$

$$\text{Now, } s = 3 + \sqrt{3} + \sqrt{2} = \left(\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}}\right)k$$

$$\Rightarrow\ 3 + \sqrt{3} + \sqrt{2} = \left(\frac{3+\sqrt{3}+\sqrt{2}}{4\sqrt{2}}\right)k \Rightarrow k = 4\sqrt{2}$$

$$\therefore\quad \text{Largest side, } c = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) \times 4\sqrt{2} = 2(\sqrt{3}+1)$$

6. **(c) :** In the word 'ASSISTANT' there are SSS AA TT I N and in 'STATISTICS" there are A C II SSS TTT. The non common letters are N, C. The identical letters are A, I, S, T.

$$\text{The probability of getting A} = \frac{^2C_1}{^9C_1} \times \frac{^1C_1}{^{10}C_1} = \frac{1}{45},$$

$$\text{probability of getting I} = \frac{^1C_1}{^9C_1} \times \frac{^2C_1}{^{10}C_1} = \frac{1}{45},$$

$$\text{probability of getting S} = \frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{10} \quad \text{and}$$

$$\text{probability of getting T} = \frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{15}$$

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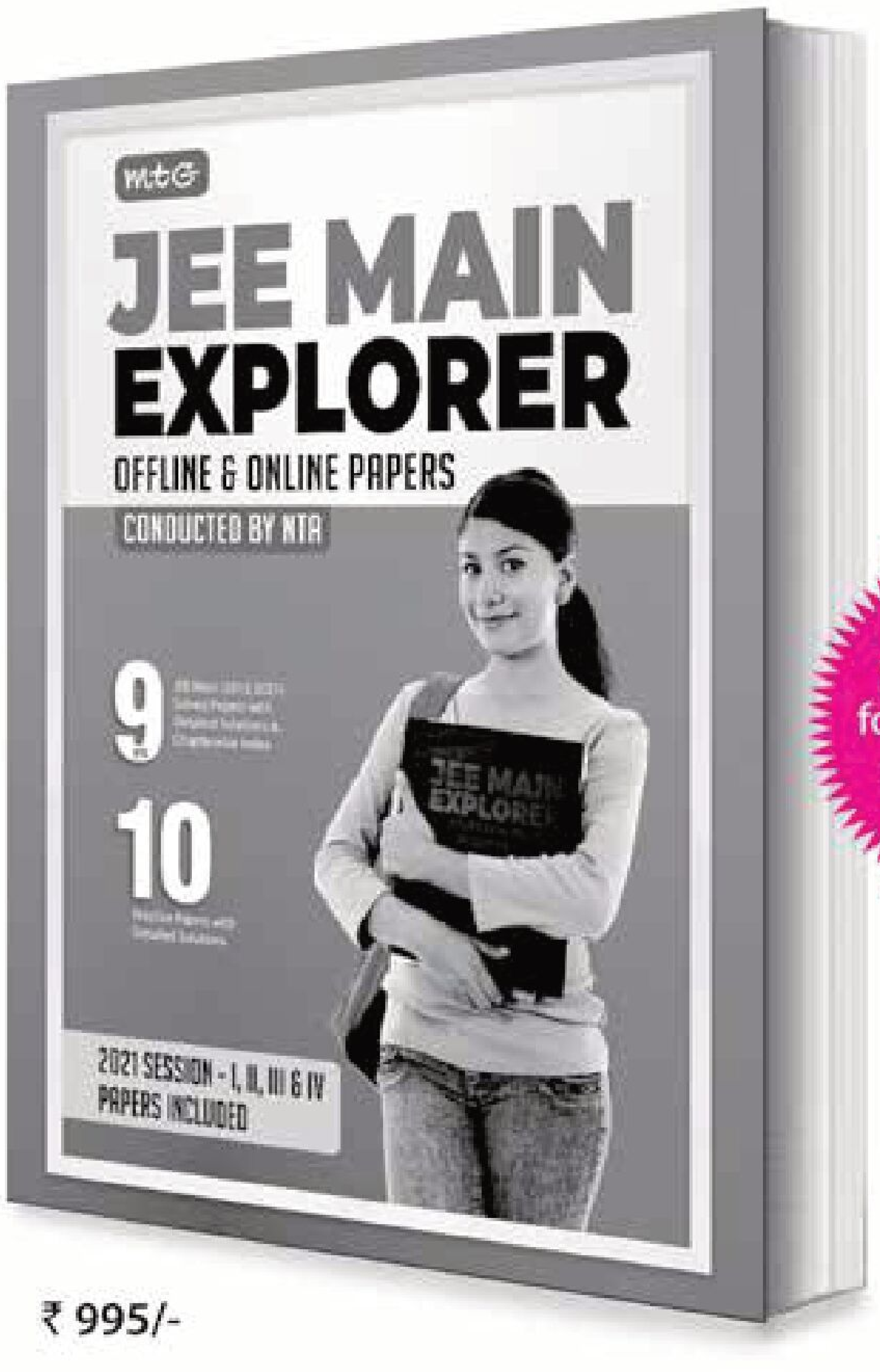
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$\therefore$ Required probability $= \frac{1}{45}+\frac{1}{45}+\frac{1}{10}+\frac{1}{15}=\frac{19}{90}$

7. (a) : We have, $|A|=\begin{vmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 3 & 2 & 1\end{vmatrix}=0+7-1=6$

$\therefore\quad (A(\text{adj}A)A^{-1})A=(A(\text{adj}A))(A^{-1}A)=(|A|I)I$

$$=|A|I=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}=2\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

8. (d): We have, $P(X\mid Y)=\frac{1}{2},\ P(Y\mid X)=\frac{1}{3}$

and $P(X\cap Y)=\frac{1}{6}$

$\therefore\ P(X)=\frac{P(X\cap Y)}{P(Y\mid X)}=\frac{1/6}{1/3}=\frac{1}{2},$

$P(Y)=\frac{P(X\cap Y)}{P(X\mid Y)}=\frac{1/6}{1/2}=\frac{1}{3}$

Clearly, X and Y are independent events.

$\therefore\quad P(X\cup Y)=P(X)+P(Y)-P(X)\cdot P(Y)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

and $P(X^C\cap Y)=P(Y)-P(X\cap Y)=\frac{1}{3}-\frac{1}{6}=\frac{1}{6}$

9. (b): Given determinant shows the volume of parallelepiped made by vectors $\vec{a},\vec{b}$ and $\vec{c}$.

$$\therefore\quad \begin{vmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{vmatrix}=|\vec{a}||\vec{b}\times\vec{c}|\sin\frac{\pi}{3}=|\vec{a}||\vec{b}||\vec{c}|\sin\frac{\pi}{3}\sin\frac{\pi}{6}$$

$$=\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\quad [\because\ |\vec{a}|=2\sqrt{2}]$$

10. (a) : Here, $p(x^2-x)+x+5=0$

$\Rightarrow\quad px^2-(p-1)x+5=0$

$\therefore\quad \alpha+\beta=\frac{p-1}{p}$ and $\alpha\beta=\frac{5}{p}$

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}\ \Rightarrow\ \frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{4}{5}$

$\Rightarrow\quad \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{4}{5}\ \Rightarrow\ \frac{(p-1)^2-10p}{5p}=\frac{4}{5}$

$\Rightarrow\quad p^2-16p+1=0\ \therefore\ p_1+p_2=16$ and $p_1p_2=1$

Now, $\frac{p_1}{p_2}+\frac{p_2}{p_1}=\frac{(p_1+p_2)^2-2p_1p_2}{p_1p_2}=\frac{256-2}{1}=254$

11. (b): The sum of the coefficients in the expansion of $(b+c)^{20}\,[1+(a-2)x]^{20}$ is given by

$S_1=(b+c)^{20}\,(a-1)^{20}$ [Putting $x=1$]

The sum of the coefficients in the expansion of $[2bcx-(b+c)y]^{10}$ is given by

$S_2=[2bc-(b+c)]^{10}$ [Putting $x=y=1$]

It is given that $S_1=S_2^2$

$\Rightarrow\quad (b+c)^{20}\,(a-1)^{20}=[2bc-(b+c)]^{20}$

$\Rightarrow\quad (b+c)(a-1)=[2bc-(b+c)]$

$\Rightarrow\quad (a-1)=\frac{2bc}{b+c}-1\ \Rightarrow\ a=\frac{2bc}{b+c}$

12. (c) : We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Putting $y=vx$, we get $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$

$\Rightarrow\quad x\frac{dv}{dx}=\frac{v}{1+v^2}-v=-\frac{v^3}{1+v^2}$

$\Rightarrow\quad \frac{1+v^2}{v^3}dv=-\frac{dx}{x}\ \Rightarrow\ \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$

$\Rightarrow\quad \ln x+\ln v-\frac{1}{2v^2}=c\ \Rightarrow\ \ln y=\frac{x^2}{2y^2}+c$

Now, $x=1,\ y=1\Rightarrow c=-\frac{1}{2}$

Again, $y=e\Rightarrow 1=\frac{x_0^2}{2e^2}-\frac{1}{2}\Rightarrow x_0^2=3e^2\ \Rightarrow\ x_0=\sqrt{3}e$

13. (a) : Let p denote the probability of correct distinction between a cup of instant coffee and a cup of percolator coffee is made.

We have, $p=75\%=0.75\quad\therefore\quad q=0.25$

$\therefore\quad P(X\geq 5)=P(X=5)+P(X=6)$

$={}^6C_5p^5q^{6-5}+{}^6C_6p^6q^{6-6}$

$={}^6C_5(0.75)^5(0.25)^1+{}^6C_6(0.75)^6\,(0.25)^0$

$=0.355957+0.177979=0.533936$

$\therefore\quad$ Chances of the claim being accepted $=0.534$

14. (c) : Let, $x_1,x_2\in R\ \therefore\ f(x_1)=f(x_2)\Rightarrow x_1|x_1|=x_2|x_2|$

$\therefore\quad$ Mapping f is one-to-one. So, $x_1=x_2$

Again, let $y\in R$ such that $y=f(x)\Rightarrow y=x|x|$

Now, when $-1\leq x\leq 1$, then $-1\leq y\leq 1$

$\therefore\quad$ Range and co-domain of f are equal.

Thus, f is onto. Therefore f is one-to-one and onto, hence f is bijective.

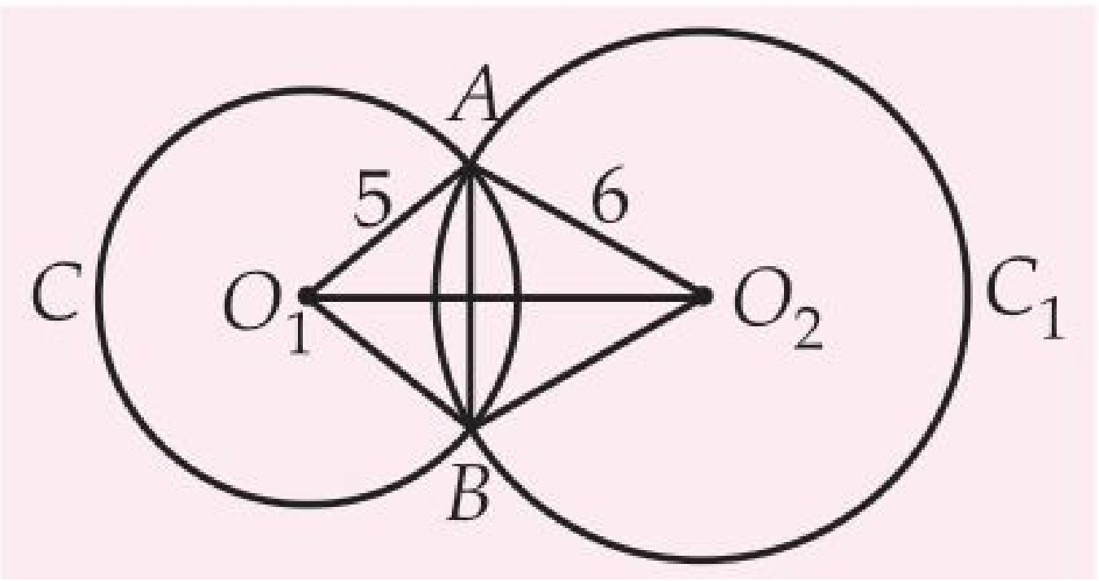
15. (c) : C : centre $O_1\equiv(1,-2)$, radius $=5$

C_1 : centre $O_2\equiv(-3,-5)$,

radius $=6$,

$O_1O_2=\sqrt{16+9}=5$

The sides of ΔO_1O_2A



are 5, 6, 5

Area of $\Delta O_1O_2A = \sqrt{8\cdot 3\cdot 2\cdot 3} = 12$

$$\Rightarrow \quad 12 = \frac{1}{2}\,\frac{AB}{2}\cdot O_1O_2 = \frac{AB}{4}\cdot 5 \quad\Rightarrow\quad AB = \frac{48}{5}$$

16. (a): We have a total 24 socks out of which 4 socks are picked up at random.

$\therefore$ Total number of ways = ${}^{24}C_4 = 10626$

$\therefore$ P (selecting atleast one pair)

= 1 – P (not selecting any pair)

$$=1-\frac{[{}^{12}C_1\times{}^{11}C_3+{}^{12}C_2\times{}^{10}C_2+{}^{12}C_3\times{}^{9}C_1+{}^{12}C_4+{}^{12}C_4]}{10626}$$

$$=1-\frac{7920}{10626}=\frac{2706}{10626}=\frac{41}{161}$$

17. (a): $x^ky^kz^k\begin{vmatrix}1 & ar & a^2r^2\\ 1 & ar^2 & a^2r^4\\ 1 & ar^3 & a^2r^6\end{vmatrix}$

$= a^{3(k+1)}\cdot r^{3(2k+1)}[(r-1)(r^4-1)-(r^2-1)^2]$

$\Rightarrow\quad k = -1$

18. (d): $y' = \dfrac{dy}{dx} = \dfrac{5(\sin^4 t\cdot\cos t+\operatorname{cosec}^5 t\cdot\cot t)}{\cos t+\operatorname{cosec} t\cot t}$

$$=\frac{5(\sin^5 t+\operatorname{cosec}^5 t)}{\sin t+\operatorname{cosec} t}$$

$$\Rightarrow\quad (y')^2=\frac{25\left[\left(\sin^5 t-\operatorname{cosec}^5 t\right)^2+4\right]}{(\sin t-\operatorname{cosec} t)^2+4}=\frac{25(y^2+4)}{(x^2+4)}$$

$\Rightarrow\quad (x^2+4)(y')^2 = 25\,(y^2+4)$

Differentiating w.r.t x and cancelling $2y'$, we get

$(x^2+4)y'' + xy' = 25y.$

19. (c): We have, $\dfrac{\pi a}{2} + 2(a+b) = L$

$$\Rightarrow\quad b=\frac{1}{2}\left[L-\frac{\pi a}{2}-2a\right]$$

Now, $f(a)=\dfrac{\pi a^2}{8}+\dfrac{3a}{2}\left[L-\dfrac{\pi a}{2}-2a\right]$

$$\Rightarrow\quad f(a) = \frac{3aL}{2}-(5\pi+24)\frac{a^2}{8}$$

$$\Rightarrow\quad f'(a) = 0 \Rightarrow a = \frac{6L}{5\pi+24}$$

$$\therefore\quad b = \frac{1}{2}\left[L-\frac{\pi a}{2}-2a\right] = \frac{(\pi+6)L}{5\pi+24}\quad\therefore\quad \frac{b}{a}=\frac{\pi+6}{6}$$

20. (d): It is linear D.E. with I.F = $\exp\int\cos x\,dx = e^{\sin x}$.

$\therefore$ The solution is given by

$y.e^{\sin x} = \int \sin x\cos x\, e^{\sin x}dx = \int te^t\,dt$ (put $t = \sin x$)

$= (t-1)e^t + c = (\sin x - 1)\,e^{\sin x} + c$

$\therefore\quad y = \sin x - 1 + ce^{-\sin x}$

Now, $y(0) = 1 \Rightarrow c = 2$, $y(x) = \sin x - 1 + 2e^{-\sin x}$

$$\therefore\quad y\left(\frac{\pi}{2}\right)=\frac{2}{e}$$

21. (1):

Box I	**Box II**
a black marbles	c black marbles
b white marbles	d white marbles

Also, $a + b + c + d = 25$

Now, $\dfrac{a}{a+b}\cdot\dfrac{c}{c+d}=\dfrac{27}{50}$

$\therefore\quad 27(a+b)(c+d) = 50\,ac$, ac is divisible by 27.

$\Rightarrow\quad ac = 27\,k \Rightarrow (a+b)(c+d) = 50\,k$

$\Rightarrow\quad (a+b)(25-(a+b)) = 50\,k$

$k = 3 \Rightarrow a + b = 15, c + d = 10, a = 9, b = 6, c = 9, d = 1$

$k = 2 \Rightarrow a + b = 20, c + d = 5, a = 18, b = 2, c = 3, d = 2$

In either case, the probability of two white marbles is

$$\frac{bd}{(a+b)(c+d)}=\frac{6}{150}=\frac{4}{100}=\frac{1}{25}=\frac{m}{n}$$

$\therefore\quad m = 1.$

22. (9): Let the angle between $\vec{a}$ and $\vec{b}$ be α and that between $\vec{a}\times\vec{b}$ and $\vec{c}$ be β.

$\therefore\ \left|[\vec{a}\,\vec{b}\,\vec{c}]\right|=6 \quad\Rightarrow\quad \sin\alpha\cos\beta = 1$

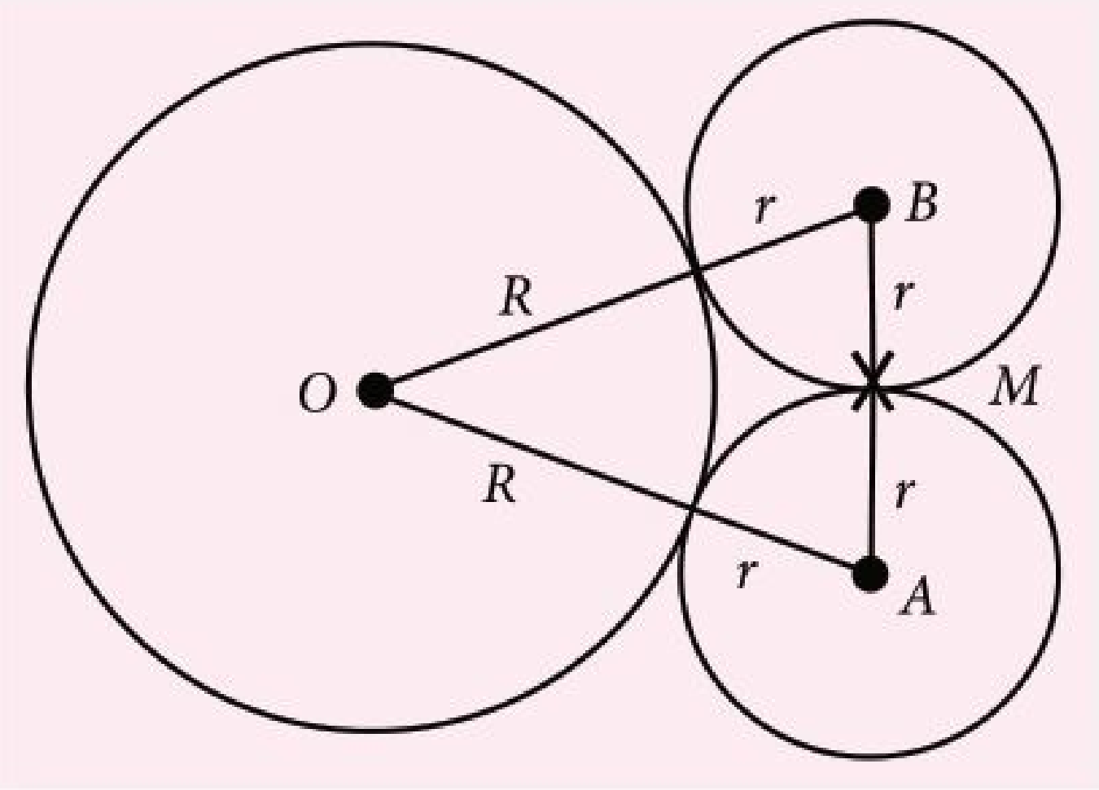
$\Rightarrow\quad \sin\alpha = 1, \cos\beta = 1 \quad\Rightarrow\quad \alpha = 90°, \beta = 0°$

So, $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular.

$$\therefore\quad \left|(\vec{a}\times\vec{c})\cdot\vec{d}\right|^2+\left|(\vec{a}\times\vec{c})\times\vec{d}\right|^2=|\vec{a}\times\vec{c}|^2\,|\vec{d}|^2$$

$=|\vec{a}|^2\,|\vec{c}|^2\,|\vec{d}|^2 = 9$

23. (8): Let the small circles of centres A and B meet at M.



$$\frac{AM}{OA}=\sin\frac{2\pi}{24}=\sin 15°$$

$$\Rightarrow\ (R+r)\,\frac{\sqrt{3}-1}{2\sqrt{2}}=r$$

$$\Rightarrow\quad \frac{R}{r}=\frac{2\sqrt{2}}{\sqrt{3}-1}\left(1-\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$=\frac{(\sqrt{3}+1)}{2}\,(2\sqrt{2}-\sqrt{3}+1)$$

$=\sqrt{6}+\sqrt{2}-1 \Rightarrow m = 6, n = 2, m + n = 8.$

24. (0): We have, $y = \sin(2\sin^{-1}x)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{2}{\sqrt{1-x^2}}\cos(2\sin^{-1}x)$$

$$\Rightarrow \quad \sqrt{1-x^2}\,\frac{dy}{dx} = 2\cos(2\sin^{-1}x)$$

$$\Rightarrow \quad (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4\{1-\sin^2(2\sin^{-1}x)\} = 4-4y^2$$

Again differentiating w.r.t. x, we get

$$2(1-x^2)\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -8y\frac{dy}{dx}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

25. (27) : The equation of the ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

$\therefore$ The eccentricity, $e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

The coordinates of the end-points of latus rectum are

$$\left(2,\frac{5}{3}\right),\left(2,-\frac{5}{3}\right),\left(-2,\frac{5}{3}\right),\left(-2,-\frac{5}{3}\right).$$

The equations of tangents at these points are

$2x + 3y - 9 = 0$ ...(i)
$-2x + 3y + 9 = 0$ ...(ii)
$-2x + 3y - 9 = 0$ ...(iii)
$2x + 3y + 9 = 0$ ...(iv)

Clearly the above tangents form a parallelogram whose area is given by

$$A = \left|\frac{\{9-(-9)\}\times\{9-(-9)\}}{\begin{vmatrix}2 & 3\\ -2 & 3\end{vmatrix}}\right| = \frac{18\times 18}{12} = 27 \text{ sq.units}$$

26. (10) : Let the volume, surface area and radius of the spherical balloon be v, s and r respectively.

$$\therefore \quad v=\frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt}=4\pi r^2\frac{dr}{dt}$$

$$\text{Again,} \quad s=4\pi r^2 \Rightarrow \frac{ds}{dt}=8\pi r\frac{dr}{dt}$$

$$\text{Now,} \quad \frac{\frac{ds}{dt}}{\frac{dv}{dt}}=\frac{2}{r} \Rightarrow \frac{ds}{dt}=\frac{2}{r}\cdot\frac{dv}{dt}=\frac{2}{7}\times 35=10$$

$$\left[\because\ r = 7 \text{ cm and } \frac{dv}{dt} = 35 \text{ cm}^3\text{/min}\right]$$

Thus, the rate of increase of the surface area is 10 cm^2/min, when diameter of the balloon is 14 cm.

27. (6) : The given equation is $3\cos x + 4\sin x = 2k + 1$

$$\Rightarrow \quad \frac{3}{\sqrt{3^2+4^2}}\cos x+\frac{4}{\sqrt{3^2+4^2}}\sin x=\frac{2k+1}{\sqrt{3^2+4^2}}$$

$$\Rightarrow \quad \frac{3}{5}\cos x+\frac{4}{5}\sin x=\frac{2k+1}{5}$$

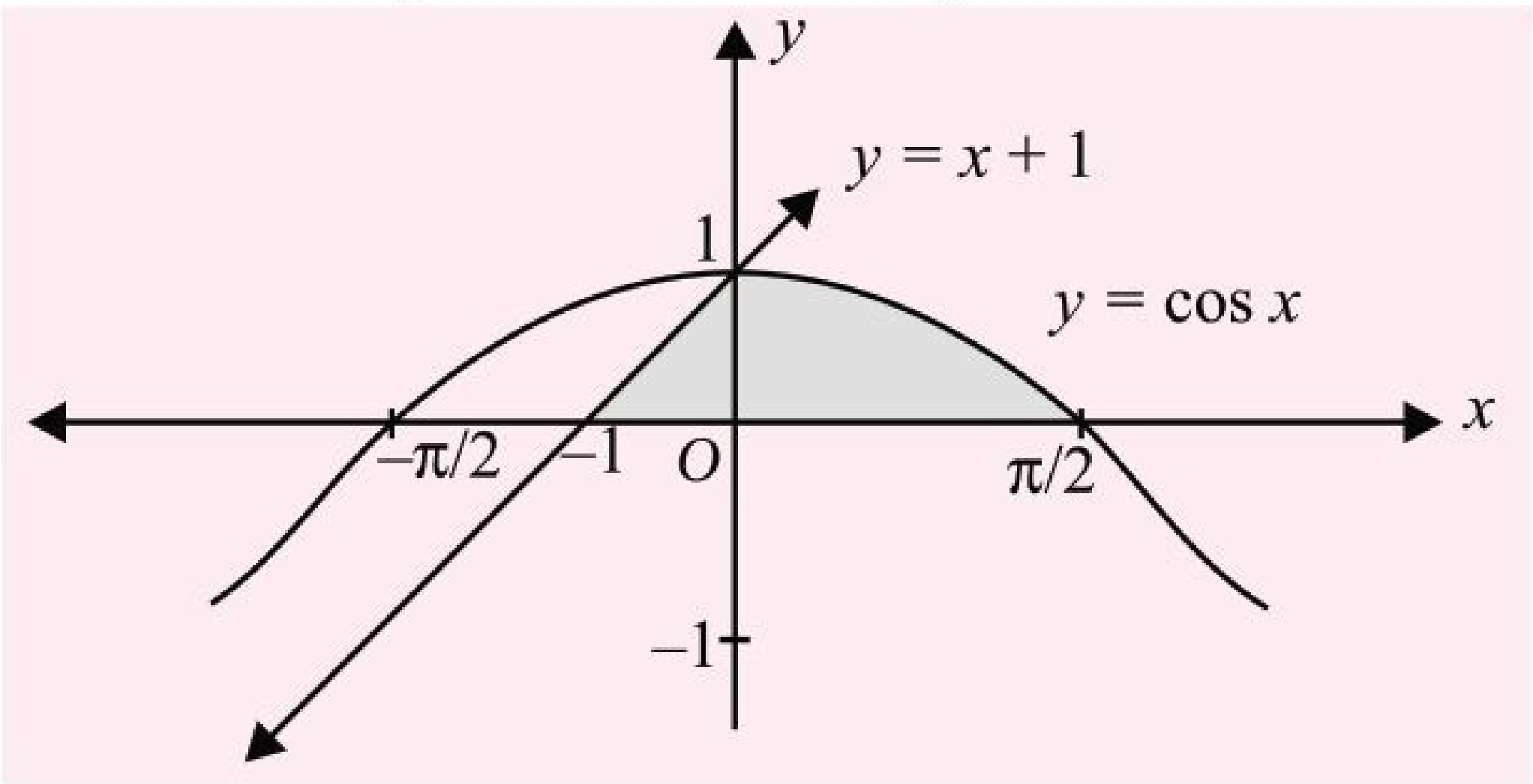
$$\Rightarrow \quad \cos(x-\alpha)=\frac{2k+1}{5} \quad \left(\text{where } \cos\alpha=\frac{3}{5}\right)$$

$$\Rightarrow \quad -1\le\frac{2k+1}{5}\le 1 \qquad [\because\ -1\le\cos(x-\alpha)\le 1]$$

$$\Rightarrow \quad -5\le 2k+1\le 5 \Rightarrow -6\le 2k\le 4 \Rightarrow -3\le k\le 2$$

Therefore integral values of k are $-3, -2, -1, 0, 1, 2$.

28. (9) : We have, $y = \cos x$ and $y = x + 1$



$$\text{The required area, } A=\int_{-1}^{0}(x+1)\,dx+\int_{0}^{\pi/2}\cos x\,dx$$

$= 1/2 + 1 = 3/2$

29. (1) : $\int \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$, $\sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}$

$$=\int\sqrt{1-x^2}\,dx=\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x+c$$

$A + B = 1/2 + 1/2 = 1.$

30. (5) : Total ways of $A \times B = 5 \times 4 = 20$

Favorable cases = (1, 8), (3, 6), (5, 4), (7, 2) ($\because a + b = 9$)

$\therefore$ Number of favourable cases = 4

$$\therefore \quad \text{Required probability}=\frac{4}{20}=\frac{1}{5}$$

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Topics to be covered : Two Dimensional Geometry and Three Dimensional Geometry

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. Equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines. Combined equation of lines that can be obtained by reflecting these lines about the x-axis is

(a) $ax^2 + 2hxy + by^2 = 0$ (b) $bx^2 + 2hxy + ay^2 = 0$

(c) $bx^2 - 2hxy + ay^2 = 0$ (d) $by^2 - 2hxy + ax^2 = 0$

2. The equation of the chord joining the points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

(a) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ (b) $\frac{x}{x_1^2+x_2^2}+\frac{y}{y_1^2+y_2^2}=1$

(c) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ (d) $\frac{x}{y_1^2+y_2^2}+\frac{y}{x_1^2+x_2^2}=1$

3. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P., with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

(a) lie on a straight line (b) lie on an ellipse

(c) lie on a circle (d) are vertices of a triangle

4. If p_1 and p_2 are the lengths of the perpendiculars from the points (2, 3, 4) and (1, 1, 4) respectively on the plane $3x - 6y + 2z + 11 = 0$, then p_1, p_2 are the roots of the equation

(a) $p^2 - 23p + 7 = 0$ (b) $7p^2 - 23p + 16 = 0$

(c) $p^2 - 17p + 16 = 0$ (d) $p^2 - 16p + 7 = 0$

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Consider the two circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 10x + 9 = 0$.

5. If the length of the common chord of the circles is $\frac{\lambda}{5}$, then λ equals _____.

6. If θ is the angle between the common tangents, then $\cos\theta = \frac{a}{b}$, where $b - a$ equals _____.

Question Stem for Question Nos. 7 and 8

Consider the three points as $A(2, 0, 3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$.

7. If the point in xy-plane which is equidistant from the given points is $(a, b, 0)$, then $a + b$ equals _____.

8. If the difference between AB and AC is $\sqrt{\lambda}-2\sqrt{2}$, then λ equals _____.

Question Stem for Question Nos. 9 and 10

AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(t^2, 2t)$.

9. If the minimum value of $|t|$ is $2\sqrt{\lambda}$, then λ equals _____.

10. If the slope of AB is $-\frac{k}{2t}$, then k equals _____.

SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*

11. If a line makes angles $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ with diagonals of a cube, then

(a) $\sum_{i=1}^{4} \cos 2\alpha_i = -\frac{4}{3}$ (b) $\sum_{i=1}^{4} \sin^2 \alpha_i = \frac{4}{3}$

(c) $\sum_{i=1}^{4} \cos^2 \alpha_i = \frac{4}{3}$ (d) $\sum_{i=1}^{4} \sin 2\alpha_i = -\frac{4}{3}$

12. The equation $f(x, y) = 3x^2 + 10xy + 8y^2 + 14x + 22y + 7 = 0$ represent a hyperbola with/whose

(a) centre at $(1, -2)$

(b) one of the asymptote is $x + 2y + 3 = 0$

(c) angle between the asymptotes is $\tan^{-1}(2/11)$

(d) equation of conjugate hyperbola is $3x^2 + 10xy + 8y^2 + 14x + 22y + 23 = 0$

13. If $P(2, 3, 1)$ is a point and $L \equiv x - y - z - 2 = 0$ is a plane, then which of the following is/are true?

(a) Origin and P lie on the same side of the plane

(b) Distance of P from the plane is $\frac{4}{\sqrt{3}}$

(c) Foot of perpendicular from point P is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$

(d) Image of point P in the plane is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$

14. If the tangent to the ellipse $9x^2 + 16y^2 = 144$ at a point $P(\theta)$ is normal to circle $x^2 + y^2 - 8x - 6y = 0$, then θ equals

(a) $\pi/6$ (b) $\pi/2$ (c) 0 (d) $-\pi/6$

15. The foot of perpendicular from $A(1, 1, 1)$ to the line joining $B(-8, 5, 6)$ and $C(12, 1, 0)$ lies on the plane(s)

(a) $2x + y + z = 10$ (b) $x - y + z = 2$

(c) $x + y - z = 2$ (d) $x + y + z = 8$

16. If the equation of directrix of the parabola $x^2 - 8x + 4y + \lambda = 0$ is $y + 2 = 0$, then

(a) $\lambda = 28$

(b) equation of axis of parabola is $x - 4 = 0$

(c) focus of parabola is $(4, -4)$

(d) vertex of parabola is $(4, -3)$ and equation of tangent at vertex is $y + 3 = 0$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$ at the ends of latus rectum. If the area of quadrilateral be α sq. units, then the value of α must be ______.

18. The equation of the plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is $x + 3y + 6z = k$, where k is ______.

19. If e_1 and e_2 are the eccentricities of hyperbolas $xy = c^2$ and $x^2 - y^2 = b^2$, then $e_1 \times e_2$ must be ______.

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. If the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ intersects the planes

$$3\beta^2 x + 3(1 - 2\alpha)y + z = 3 = -\frac{1}{2}\{6\alpha^2 x + 3(1 - 2\beta)y + 2z\},$$

then point $(\alpha, \beta, 1)$ lies on the plane

(a) $2x - y + z = 4$ (b) $x + y - z = 2$

(c) $x - 2y = 0$ (d) $2x - y = 0$

2. *ABCD* is a square of side 1 unit. A circle passes through vertices A, B of the square and the remaining two vertices of the square lie outside the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

(a) $\sqrt{5}$ (b) $\frac{1}{2}\sqrt{10}$ (c) $\frac{1}{3}\sqrt{12}$ (d) $\sqrt{8}$

3. The direction cosines of two lines are connected by relations $l + m + n = 0$ and $2lm + 2ln - mn = 0$, then

(a) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2} = -3/2$

(b) $l_1 l_2 + m_1 m_2 + n_1 n_2 = -1/2$

(c) $l_1 m_1 n_1 + l_2 m_2 n_2 = -\sqrt{6}/9$

(d) $(l_1 + l_2)(m_1 + m_2)(n_1 + n_2) = \frac{\sqrt{6}}{18}$

4. In the *xy*-plane, how many straight lines whose *x*-intercept is a prime number and whose *y*-intercept is a positive integer, pass through the point (4, 3)?

(a) 1 (b) 2 (c) 3 (d) 4

5. Which of the following points lie on the line passing through (1, 2, 3) and parallel to the planes $\vec{r}\cdot(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}\cdot(3\hat{i} + \hat{j} + \hat{k}) = 6$?

(a) $(-2, 7, 7)$ (b) $(4, -3, -1)$

(c) $(-5, 12, 11)$ (d) $(1, 2, 3)$

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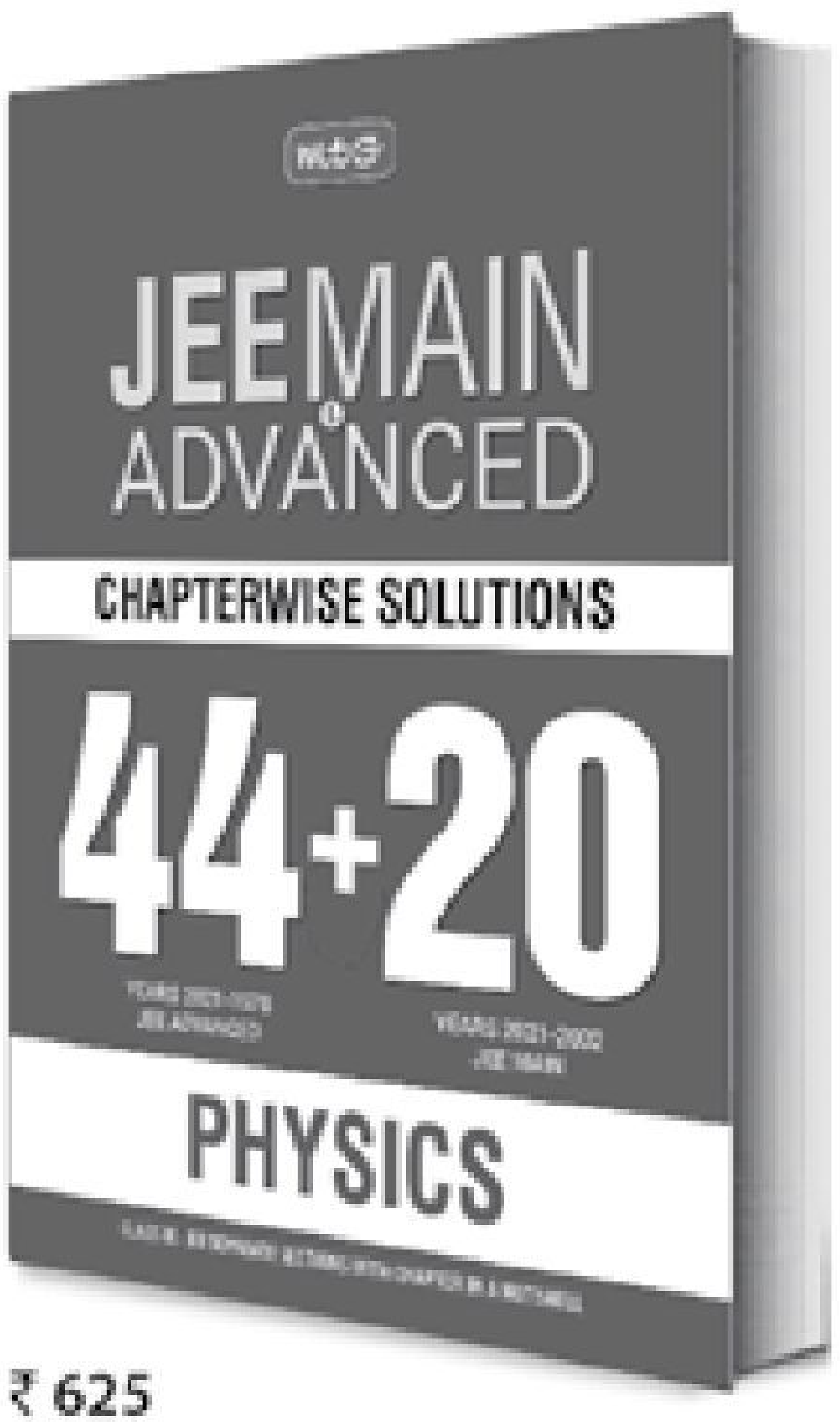
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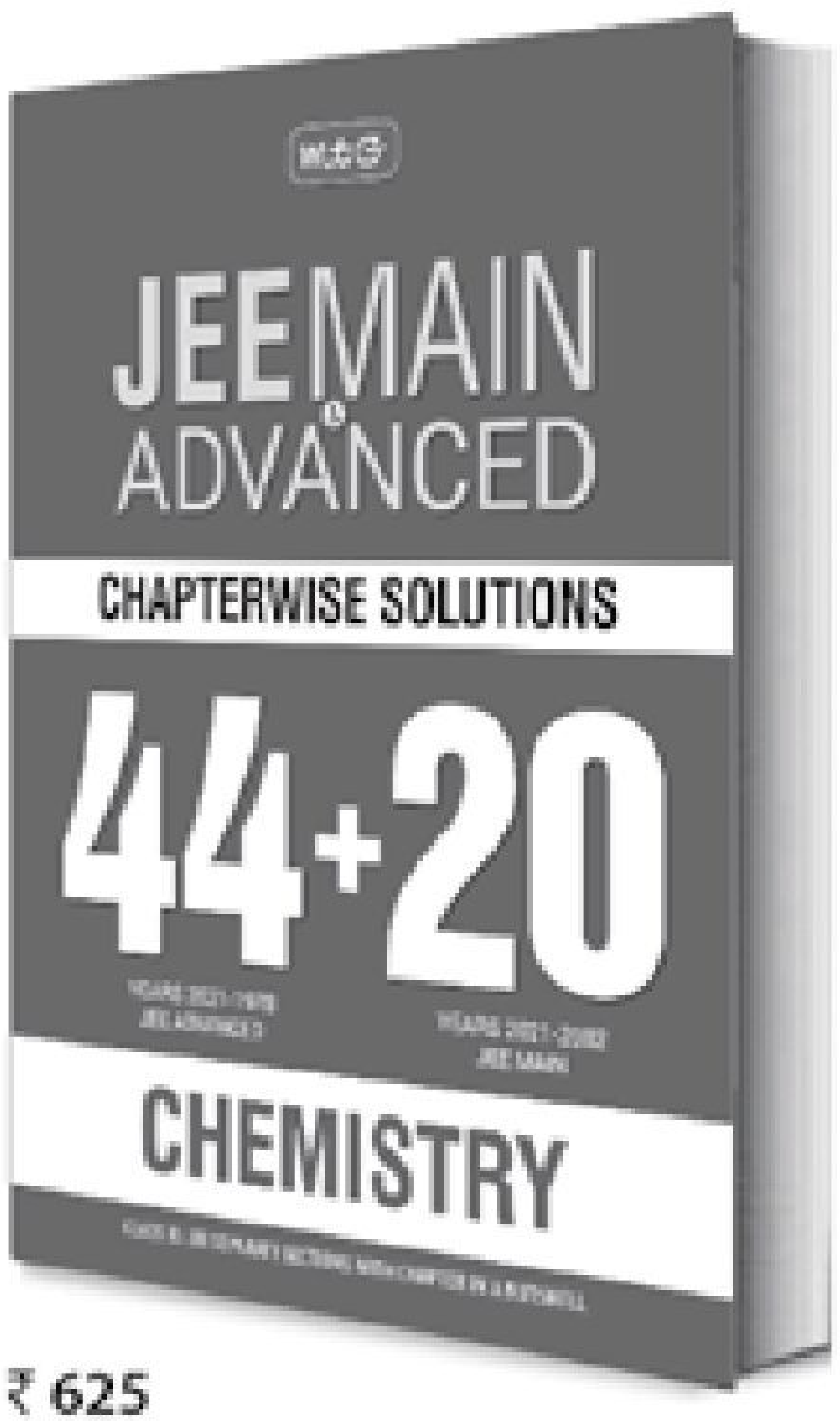
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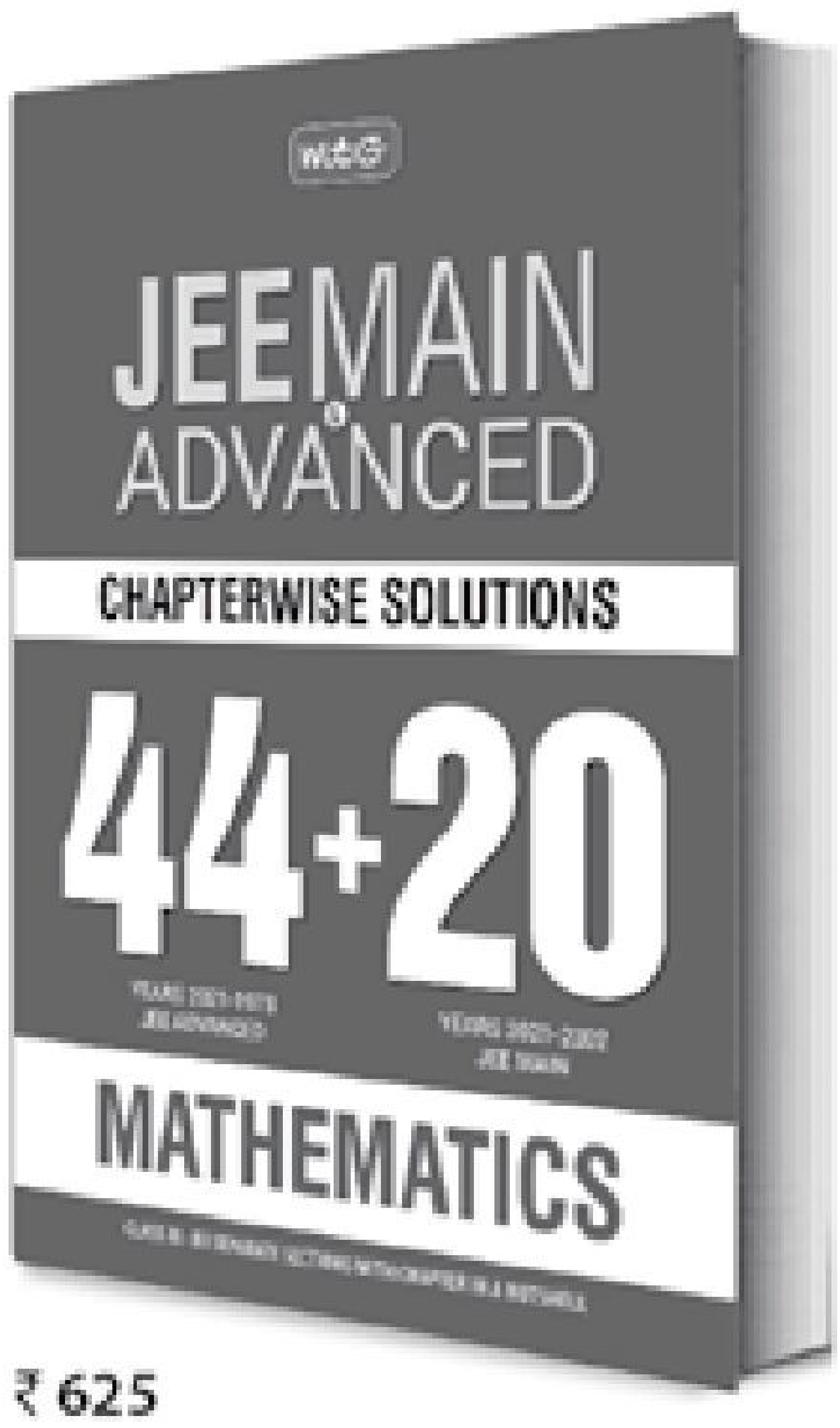
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6. If $P \equiv (1, 0)$, $Q \equiv (-1, 0)$ and $R \equiv (2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is

(a) a straight line parallel to x-axis

(b) a circle passing through the origin.

(c) a circle with the centre at the origin

(d) a straight line parallel to y-axis.

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

Consider, the vertices of a tetrahedron as $A(0, 0, 0)$, $B(3, 0, 0)$, $C(2, 3, 0)$ and $D(1, 1, 1)$.

7. If the distance of B from the plane ACD is $\frac{9}{\sqrt{\lambda}}$, then λ equals ______.

8. If the volume of the tetrahedron is $\frac{a}{b}$, then $a + b$ equals ______.

Question Stem for Question Nos. 9 and 10

Let P be a variable point on the ellipse $x^2 + 2y^2 = 2$ and S, S_1 be the foci.

9. The locus of the centroid of triangle PSS_1 is a conic of eccentricity $e = \frac{1}{\sqrt{\lambda}}$, then λ equals ______.

10. The locus of the incentre I of triangle PSS_1 is a conic of eccentricity $e = \sqrt{k\sqrt{k} - k}$, then $\frac{k}{2}$ equals ______.

Question Stem for Question Nos. 11 and 12

Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$,

$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

11. If the unit vector perpendicular to both L_1 and L_2 is $\frac{-\hat{i} - 7\hat{j} + \lambda\hat{k}}{\lambda\sqrt{3}}$, then λ equals ______.

12. The shortest distance between L_1 and L_2 is $\frac{k}{5\sqrt{3}}$, then k equals ______.

SECTION 3

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*

Paragraph for Q. 13-14

Consider the triangle with vertices $A(8, 0)$, $B(0, 6)$ and $C(0, 0)$. Equilateral triangles BCD, CAE, ABF are constructed all externally on the sides of the triangle. Let G_1, G_2, G_3 be the centroids of the triangles BCD, CAE, ABF respectively.

13. If $F = (x, y)$, then $x - y =$

(a) 1 (b) $\sqrt{3}$ (c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

14. $G_2 G_3 =$

(a) $\frac{2}{\sqrt{3}}\left(25 + 12\sqrt{3}\right)^{1/2}$ (b) $\frac{4}{\sqrt{3}}\left(25 + 12\sqrt{3}\right)^{1/2}$

(c) $\frac{2}{\sqrt{3}}\left(12 + 25\sqrt{3}\right)^{1/2}$ (d) $\frac{4}{\sqrt{3}}\left(12 + 25\sqrt{3}\right)^{1/2}$

Paragraph for Q. 15-16

Let A and B be the feet of perpendiculars drawn from a point P to the lines $\frac{x}{2} = \frac{y}{1} = \frac{z-1}{0}$ and $\frac{x}{2} = \frac{y}{-1} = \frac{z+1}{0}$ respectively.

15. If $\angle AOB = \frac{\pi}{2}$, where O is the origin, then the locus of P is

(a) $3(4x^2 - y^2) = 25$ (b) $3(4x^2 + y^2) = 25$

(c) $3(x^2 - 4y^2) = 25$ (d) $3(x^2 + 4y^2) = 25$

16. If $\angle APB = \frac{\pi}{2}$, then the locus of P is

(a) $3(x^2 - 4y^2) = 25(1 - z)^2$

(b) $3(x^2 - 4y^2) = 25\,(z^2 - 1)$

(c) $3(4x^2 - y^2) = 25\,(1 - z^2)$

(d) $3(4x^2 - y^2) = 25\,(z^2 - 1)$

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. When the origin is shifted to $(2, -3)$, the equation $y^2 + 8x + 6y - 7 = 0$ reduces to $y^2 = 4ax$. The absolute value of 'a' is equal to ______.

18. If the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles, then the value of $[p]$, where $[\cdot]$ denotes the greatest integer function, is ______.

19. The equation of hyperbola whose asymptotes are $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ which passes through the point $(0, 1)$ is $12x^2 - 7xy - 12y^2 + 31x + 17y + A = 0$ and the equation of conjugate hyperbola is $12x^2 - 7xy - 12y^2 + ax + by + c = 0$, then the sum of the digits of $a + b + c$ is ______.

SOLUTIONS

PAPER-1

1. (d) : Let the lines represented by $ax^2 + 2hxy + by^2 = 0$ be $y = m_1x$ and $y = m_2x$, then $m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = a/b$. If these lines are reflected about the x-axis, then their equation becomes $y + m_1x = 0$ and $y + m_2x = 0$ and their combined equation is $(y + m_1x)(y + m_2x) = 0$

$\Rightarrow$ $y^2 + xy(m_1 + m_2) + m_1m_2x^2 = 0$

$\Rightarrow$ $by^2 - 2hxy + ax^2 = 0$

2. (a) : We know that the chord of $xy = c^2$ whose midpoint is (h, k) is $\frac{x}{h}+\frac{y}{k}=2$ ...(i)

Also, $(h,k)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Now, by (i), we have

$$\frac{x}{\frac{x_1+x_2}{2}}+\frac{y}{\frac{y_1+y_2}{2}}=2 \Rightarrow \frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$$

3. (a) : Since x_1, x_2, x_3 and y_1, y_2, y_3 are in G.P

$\Rightarrow$ $\frac{x_2}{x_1}=\frac{x_3}{x_2}=k_1$ and $\frac{y_2}{y_1}=\frac{y_3}{y_2}=k_1$

$$\because \text{ Area} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Hence the points are collinear.

4. (b) : We have

$$p_1 = \frac{3\times2-6\times3+2\times4+11}{\sqrt{3^2+(-6)^2+(2)^2}} = \frac{7}{7} = 1$$

and $$p_2 = \frac{3\times1-6\times1+2\times4+11}{\sqrt{3^2+(-6)^2+(2)^2}} = \frac{16}{7}$$

$\therefore$ p_1, p_2 are the roots of the equation.

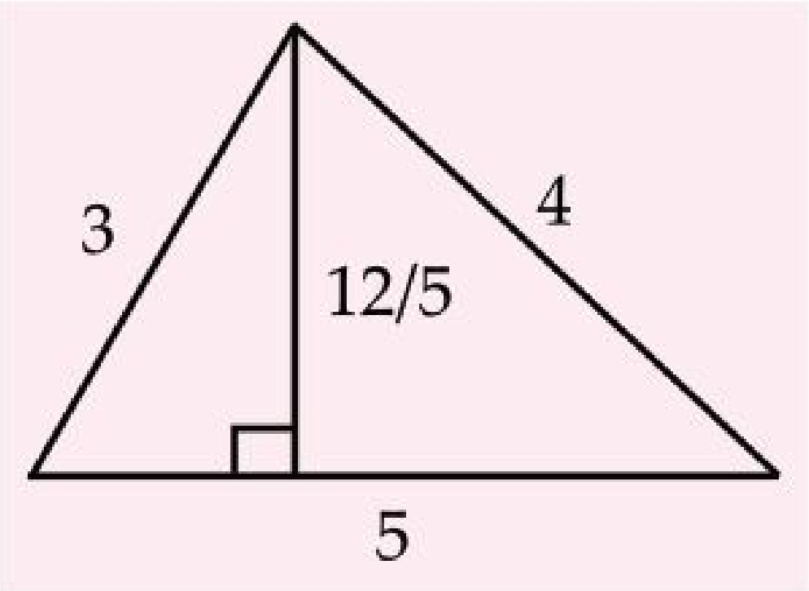
$$p^2 - \left(1+\frac{16}{7}\right)p + \frac{16}{7} = 0$$

$\Rightarrow$ $7p^2 - 23p + 16 = 0$

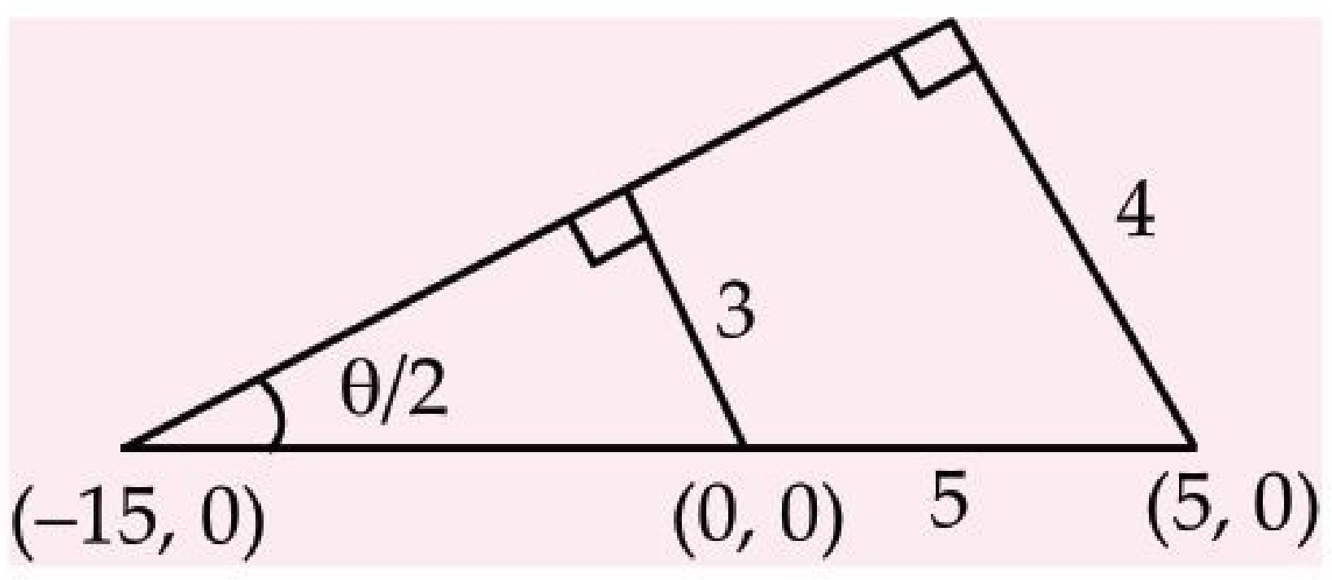
5. (24) : The circles are orthogonal.

Length of common chord is

$2 \cdot \frac{12}{5} = \frac{24}{5}$.



6. (2) : Since, the tangents meet at $(-15, 0)$.



$\therefore$ $\sin\frac{\theta}{2} = \frac{3}{15} = \frac{1}{5}$

and $\cos\theta = 1-2\sin^2\frac{\theta}{2} = \frac{23}{25}$.

7. (5) : Let the point in xy plane be $P(x, y, 0)$.

$\therefore$ $PA = PB = PC$

$$\Rightarrow \sqrt{(x-2)^2+y^2+9} = \sqrt{x^2+(y-3)^2+4} = \sqrt{x^2+y^2+1}$$

Equating first and third, we get

$(x-2)^2 + y^2 + 9 = x^2 + y^2 + 1 \Rightarrow x = 3$

Equating second and third, we get

$x^2 + (y-3)^2 + 4 = x^2 + y^2 + 1 \Rightarrow y = 2$

$\therefore$ Required point, $P \equiv (3, 2, 0) \equiv (a, b, 0)$

$\therefore$ $a + b = 5$

8. (14) : $AB = \sqrt{4+9+1} = \sqrt{14}$

$AC = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\therefore$ $AB - AC = \sqrt{14} - 2\sqrt{2} = \sqrt{\lambda} - 2\sqrt{2} \Rightarrow \lambda = 14$

9. (2) : The normals at $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$ meet at $C(t^2, 2t)$.

$\therefore$ $t = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2} \Rightarrow t_1t_2 = 2,\ t_1 + t_2 = -t$

Now, $(t_1 + t_2)^2 \geq 4t_1t_2$

$\Rightarrow$ $t^2 \geq 8$, $|t| \geq 2\sqrt{2}$

$\therefore$ Minimum value of $|t| = 2\sqrt{2} = 2\sqrt{\lambda} \Rightarrow \lambda = 2$

10. (4) : The slope of $AB = \frac{2}{t_1+t_2} = \frac{-2}{t} = \frac{-k}{2t}$

$\Rightarrow$ $k = 4$

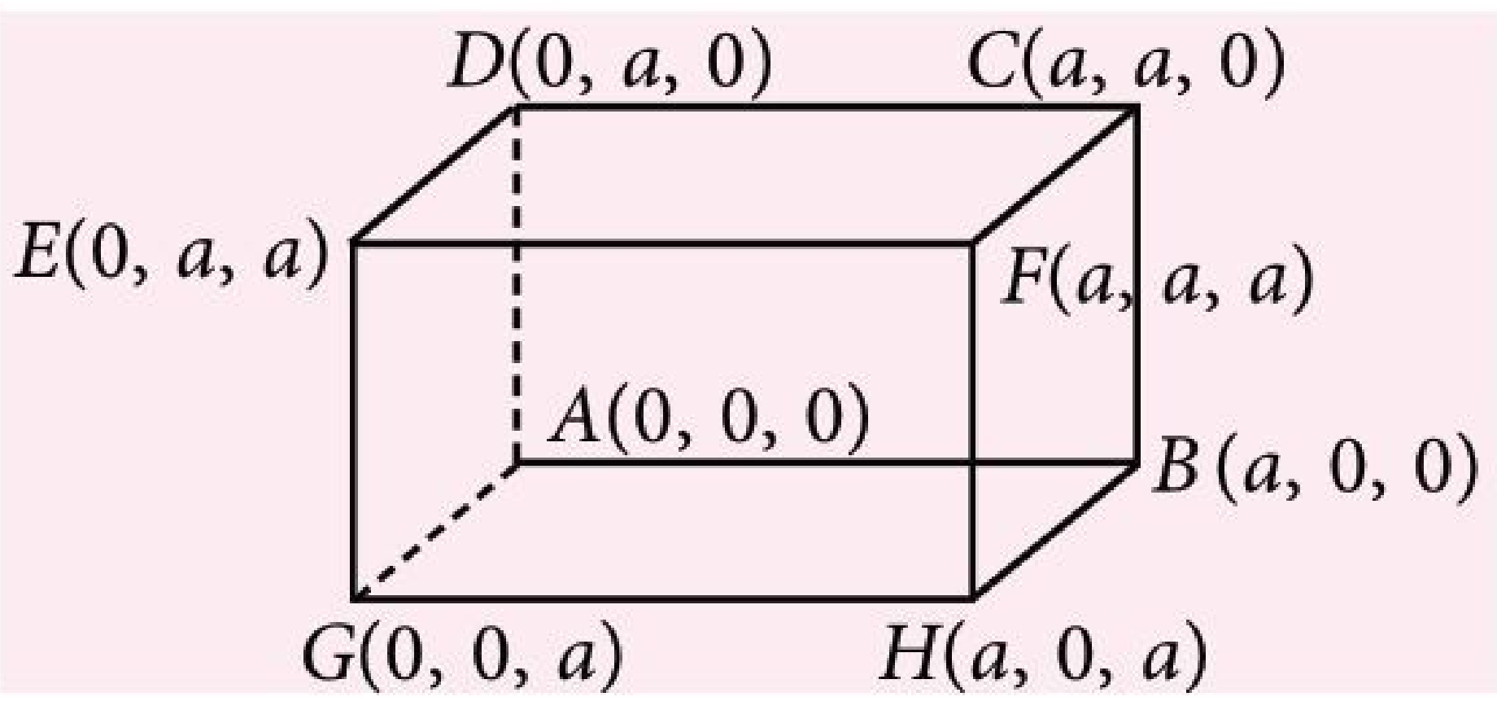
11. (a, c) : Diagonals

$AF = a\hat{i} + a\hat{j} + a\hat{k}$

$BE = -a\hat{i} + a\hat{j} + a\hat{k}$

$CG = a\hat{i} + a\hat{j} - a\hat{k}$

$HD = a\hat{i} - a\hat{j} + a\hat{k}$



where l, m, n are direction cosines of line

$$\cos\alpha_1 = \frac{al+am+an}{\sqrt{3}a} = \frac{l+m+n}{\sqrt{3}} \quad \text{...(i)}$$

$$\cos\alpha_2 = \frac{-al+ma+na}{\sqrt{3}a} = \frac{-l+m+n}{\sqrt{3}} \quad \text{...(ii)}$$

$$\cos\alpha_3 = \frac{al+ma-na}{\sqrt{3}a} = \frac{l+m-n}{\sqrt{3}} \quad \text{...(iii)}$$

$$\cos\alpha_4 = \frac{al-ma+na}{\sqrt{3}a} = \frac{l-m+n}{\sqrt{3}} \quad \text{...(iv)}$$

Squaring and then adding (i), (ii), (iii) and (iv), we get $\cos^2\alpha_1 + \cos^2\alpha_2 + \cos^2\alpha_3 + \cos^2\alpha_4$

$$= \frac{1}{3}[4l^2 + 4m^2 + 4n^2] = \frac{4}{3}(l^2 + m^2 + n^2) = \frac{4}{3}$$

Also, $\cos 2\alpha_1 + \cos 2\alpha_2 + \cos 2\alpha_3 + \cos 2\alpha_4$
$= 2(\cos^2\alpha_1 + \cos^2\alpha_2 + \cos^2\alpha_3 + \cos^2\alpha_4) - 4$

$$= 2\cdot\frac{4}{3} - 4 = -\frac{4}{3}.$$

12. (a, b, c, d) : Let equation of asymptotes of hyperbola $3x^2 + 10xy + 8y^2 + 14x + 22y + 7 = 0$ is given by
$3x^2 + 10xy + 8y^2 + 14x + 22y + \lambda = 0$ ...(i)
Now, (i) is combined equation of asymptotes *i.e.*, (i) represents pair of straight line.
Then, $\Delta = 0$ *i.e.*, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
$\Rightarrow$ $3 \cdot 8 \cdot \lambda + 2(7)(11)(5) - 3(121) - 8(49) - 25\lambda = 0$
$\Rightarrow$ $\lambda = 15$
$\therefore$ Combined equation of asymptotes is
$3x^2 + (10y + 14)x + (8y^2 + 22y + 15) = 0$

$$\therefore\ x = \frac{-(10y+14) \pm \sqrt{4y^2 + 16y + 16}}{6}$$

$\Rightarrow$ $3x = -(5y + 7) \pm (y + 2)$
$\therefore$ Equation of asymptotes are $3x + 4y + 5 = 0$ and $x + 2y + 3 = 0$
Now, the point of intersection of these asymptotes is the centre of hyperbola *i.e.*, $x = 1$, $y = -2$
$\therefore$ Centre is $(1, -2)$
Also, equation of conjugate hyperbola = 2(combined equation of asymptotes) – (equation of given hyperbola)
$\therefore$ Conjugate hyperbola is $2(3x^2 + 10xy + 8y^2 + 14x + 22y + 15) - (3x^2 + 10xy + 8y^2 + 14x + 22y + 7) = 0$
$\Rightarrow$ $3x^2 + 10xy + 8y^2 + 14x + 22y + 23 = 0$
Again, angle between the asymptotes (θ), is given by

$$\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right) = \tan^{-1}\left(\frac{2}{11}\right)$$

13. (a,b,c) : (a) Since, (0, 0, 0) and (2, 3, 1) gives same sign for the expression $x - y - z - 2$ hence, lies on same side.

(b) Distance of P from plane $= \dfrac{|2-3-1-2|}{\sqrt{3}} = \dfrac{4}{\sqrt{3}}$

(c) Equation of a line passing through $P(2, 3, 1)$ and perpendicular to the plane $x - y - z - 2 = 0$ is

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda$$

$\therefore$ General point = $(\lambda + 2, 3 - \lambda, 1 - \lambda)$
and it is lies on the plane, then

$$\lambda + 2 - (3 - \lambda) - (1 - \lambda) - 2 = 0 \Rightarrow \lambda = \frac{4}{3}$$

$$\therefore \text{ Foot of perpendicular} = \left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$$

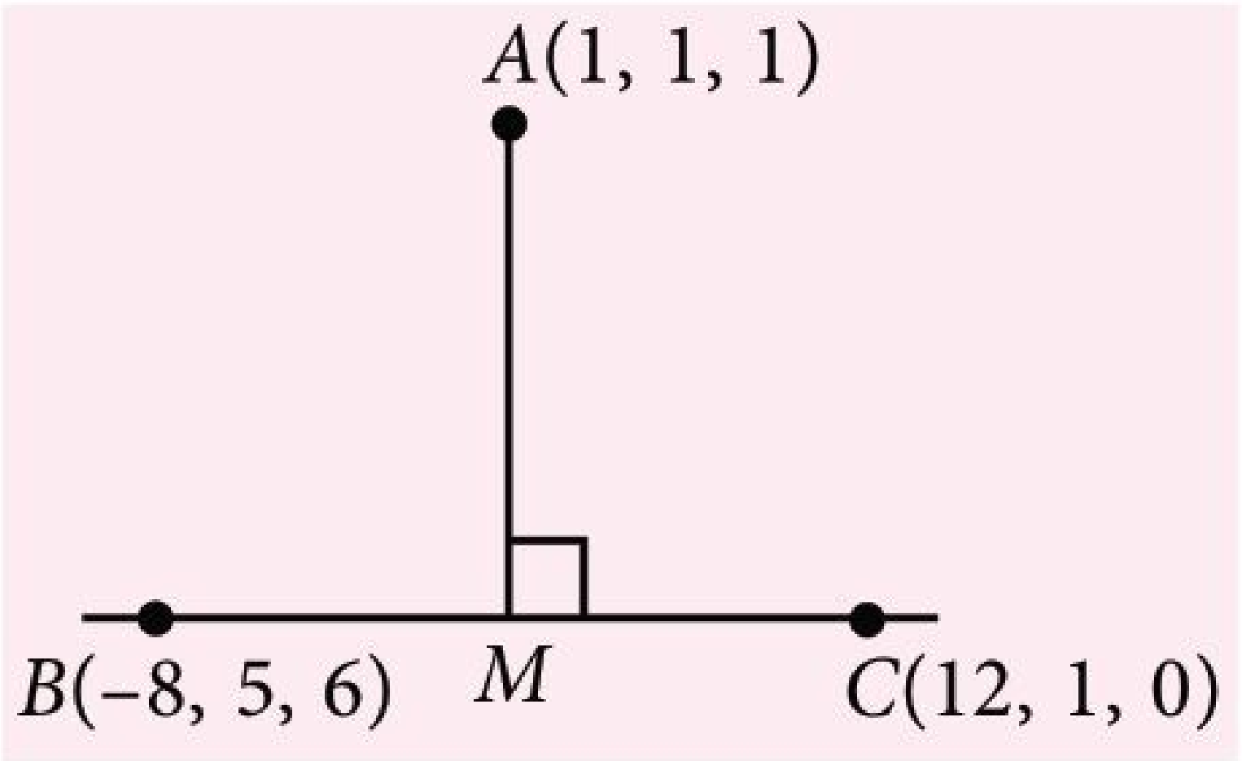
14. (b, c) : Here, equation of ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$
Now, equation of tangent at $P(\theta) = P(4\cos\theta, 3\sin\theta)$ is given by $3x\cos\theta + 4y\sin\theta = 12$ ...(i)
Now, (i) passes through the centre (4, 3).
$\therefore$ $12\cos\theta + 12\sin\theta = 12$
$\Rightarrow$ $\cos\theta + \sin\theta = 1 \Rightarrow \theta = 0$ or $\pi/2$

15. (a, b, c, d) : Equation of BC is given by

$$\vec{r} = (-8\hat{i} + 5\hat{j} + 6\hat{k}) + \lambda(20\hat{i} - 4\hat{j} - 6\hat{k})$$

$$\Rightarrow \vec{r} = (-8 + 20\lambda)\hat{i} + (5 - 4\lambda)\hat{j} + (6 - 6\lambda)\hat{k}$$

Coordinate of M is
$(-8 + 20\lambda, 5 - 4\lambda, 6 - 6\lambda)$



Now, $\overrightarrow{AM} \cdot \overrightarrow{BC} = 0$
$\Rightarrow$ $20(-9 + 20\lambda) - 4(4 - 4\lambda) - 6(5 - 6\lambda) = 0$
$\Rightarrow$ $\lambda = \dfrac{1}{2}$
$\therefore$ Coordinates of M is (2, 3, 3).

16. (a, b, c, d) : We have, $x^2 - 8x + 4y + \lambda = 0$

$$\Rightarrow (x-4)^2 = -4\left(y + \frac{\lambda - 16}{4}\right) \quad \text{...(i)}$$

$\Rightarrow$ $X^2 = 4(-1)Y$, where $X = x - 4$, $Y = \left(y + \dfrac{\lambda - 16}{4}\right)$
$\therefore$ Equation of directrix is $Y = 1$

$$i.e.,\ y + \frac{\lambda - 16}{4} = 1 \Rightarrow\ y + 2 = 3 + \frac{16 - \lambda}{4} = 0 \Rightarrow \lambda = 28$$

$\therefore$ Equation (i) reduces to $(x - 4)^2 = -4(y + 3)$
Thus, vertex $(4, -3)$ and focus, $X = 0$ and $Y = -1$ *i.e.*, $(4, -4)$.

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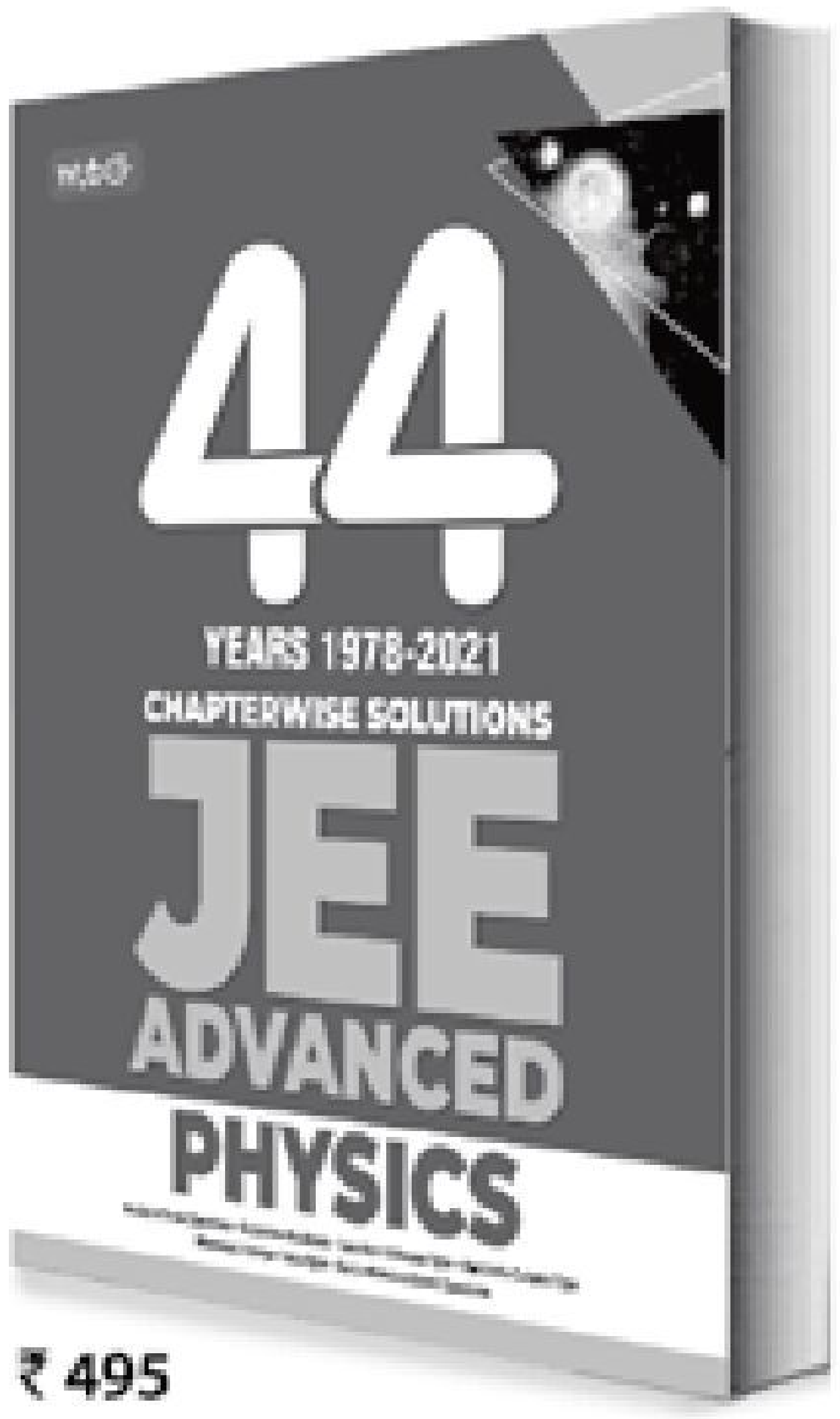

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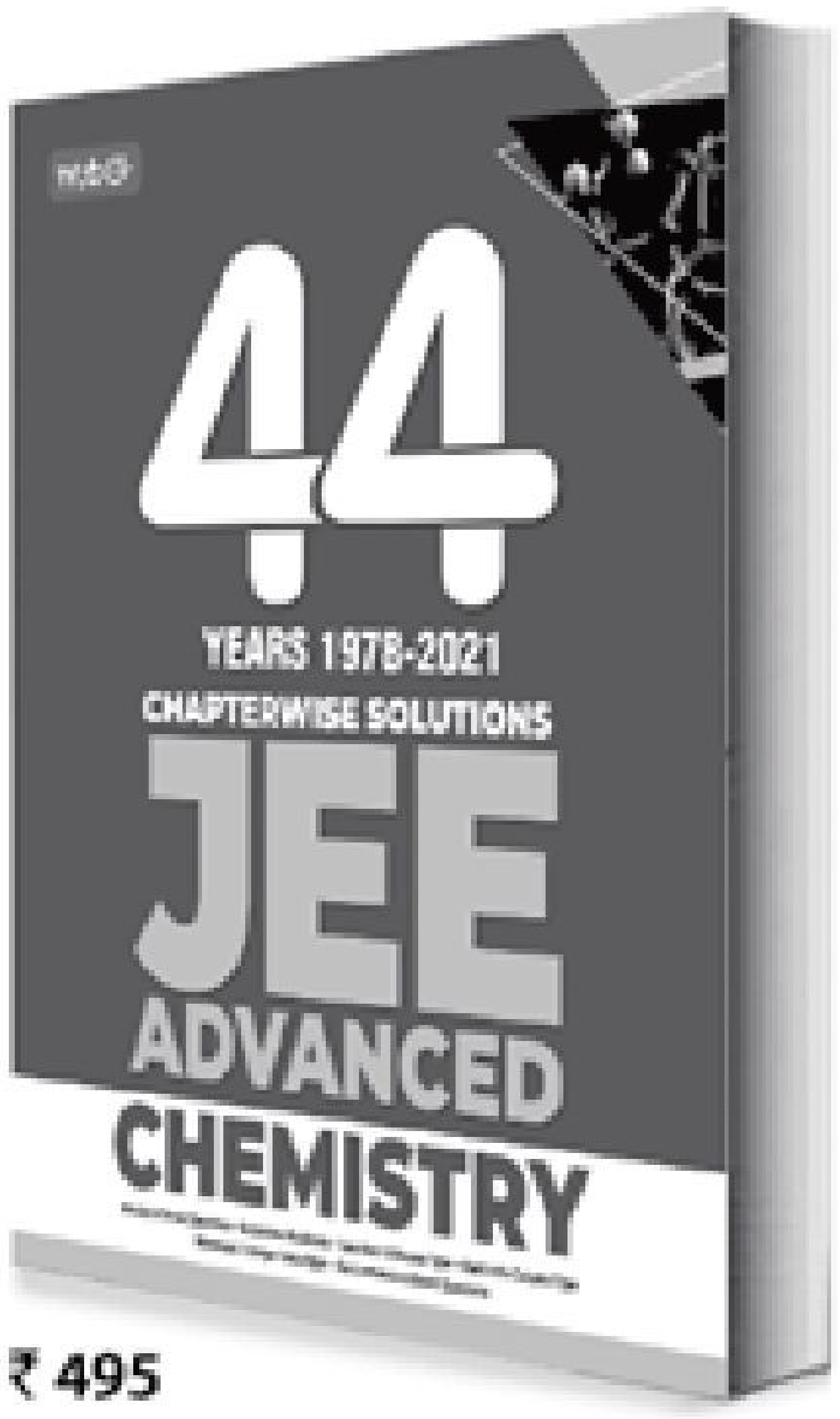

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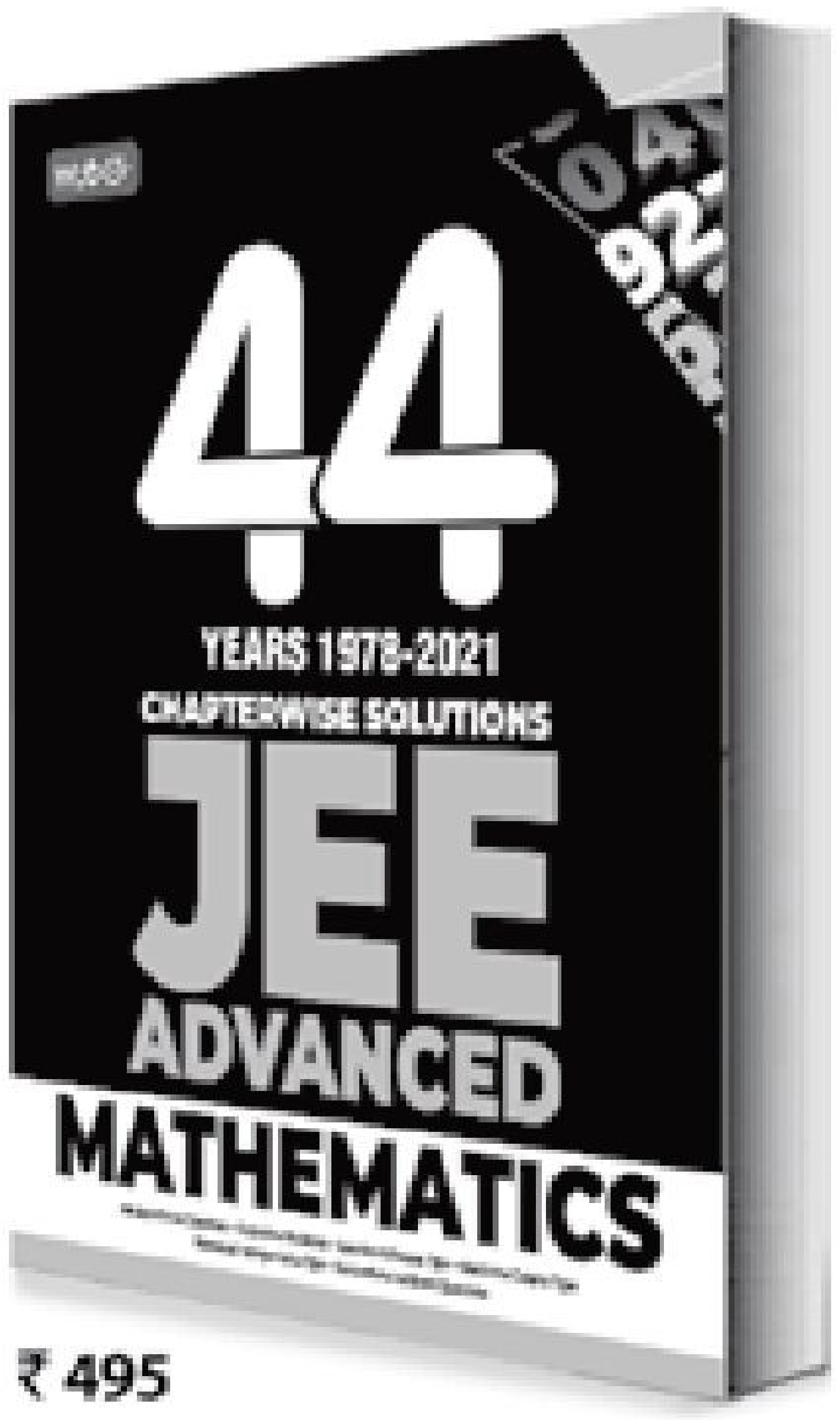

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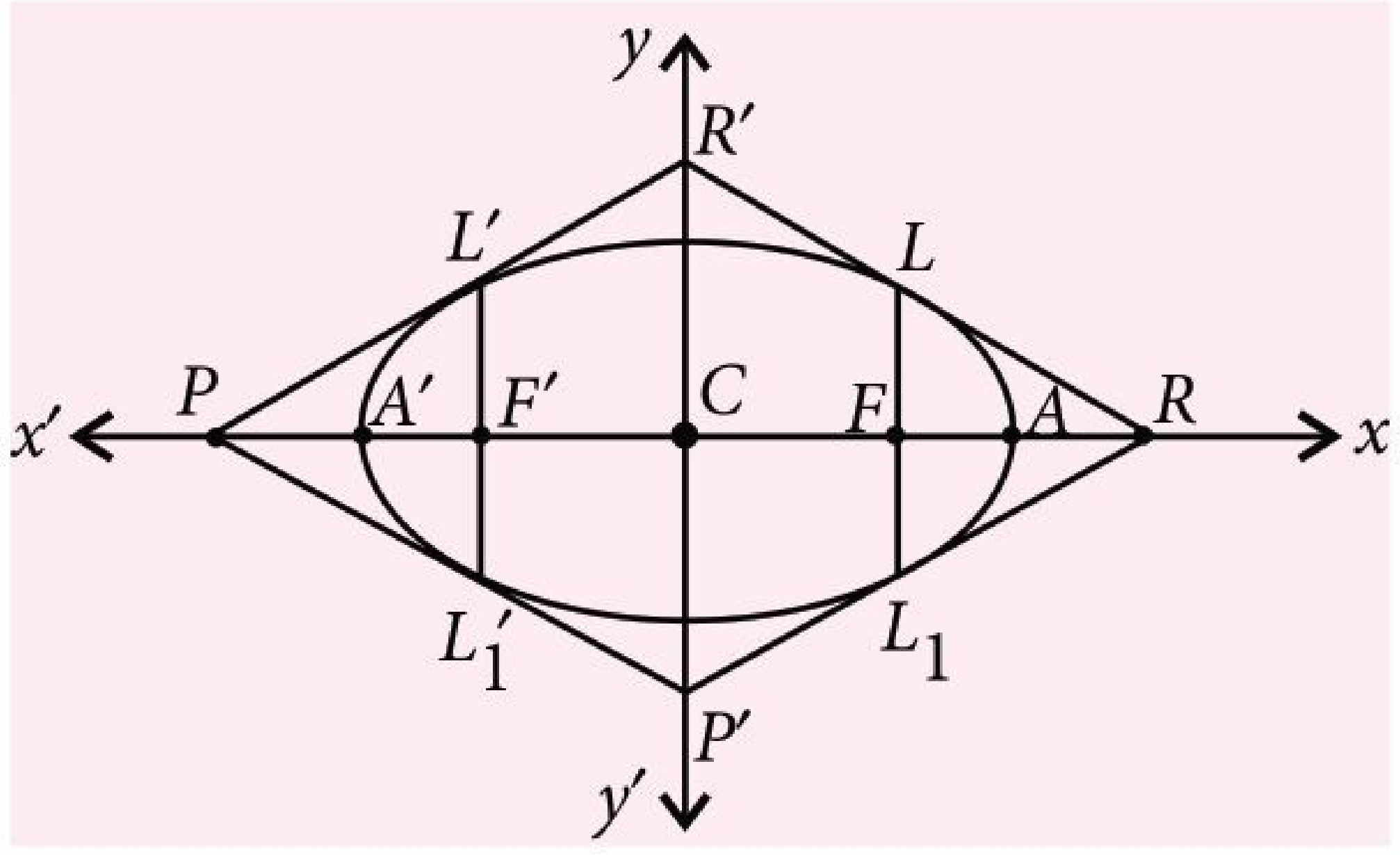
17. (54) :



The given equation of ellipse is $\frac{x^2}{9}+\frac{y^2}{8}=1$

$\Rightarrow \quad a^2=9,\ b^2=8 \ \Rightarrow e=\frac{1}{3}$ (eccentricity)

Now, $L\left(ae,\ \frac{b^2}{a}\right)=L\left(1,\ \frac{8}{3}\right)$

$\therefore$ Equation of tangent at $\left(1,\ \frac{8}{3}\right)$ is $\frac{x}{9}+\frac{y}{3}=1$ which meets x axis and y axis at $R(9, 0)$ and $R'(0, 3)$.

$\therefore \quad CR = 9$ and $CR' = 3$

Now, area of $\Delta\ CRR'=\frac{1}{2}CR\times CR'=\frac{1}{2}\times 9\times 3=\frac{27}{2}$

$\therefore$ Area of quadrilateral $PP'RR'$ = 4 × Area of $\Delta CRR'$

$=4\times\frac{27}{2}=54$ sq.units $\quad\therefore\ \alpha = 54$

18. (7) : Equation of plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ is

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

$\Rightarrow (2 + \lambda)x + (-5 + \lambda)y + (1 + 4\lambda)\, z -3 -5\lambda = 0$ ...(i)

which is parallel to the plane $x + 3y + 6z = 1$.

Then, $\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6} \quad \therefore \lambda=\frac{-11}{2}$

From (i), we have, $-\frac{7}{2}x-\frac{21}{2}y-21z+\frac{49}{2}=0$

$\therefore \quad x + 3y + 6z = 7$. Hence, $k = 7$

19. (2) : Since, both $xy = c^2$ and $x^2 - y^2 = b^2$ are rectangular hyperbola.

$\therefore \quad e_1=\sqrt{2}=e_2 \quad \Rightarrow \quad e_1 \times e_2 = 2$

PAPER-2

1. (a, b, c) : Any point $(\lambda, 2\lambda, 3\lambda)$ on line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ intersects the planes $3\beta^2x + 3(1 - 2\alpha)y + z - 3 = 0$ and $\frac{1}{2}\{6\alpha^2x+3(1-2\beta)y+2z\}+3=0$ at same point.

$$\therefore \quad \frac{3}{3\beta^2+6(1-2\alpha)+3}=\frac{-6}{6\alpha^2+6(1-2\beta)+6}$$

$\Rightarrow \quad 2(\beta - 1)^2 + 3(\alpha - 2)^2 = 0 \Rightarrow \alpha = 2, \beta = 1$

2. (b) : Let $A = (0, 1), B = (0, 0), C = (1, 0)$ and $D = (1, 1)$

Family of circles passing through A, B is

$x^2 + y^2 - y + \lambda x = 0$

Also, $\sqrt{1+\lambda}=2 \ \Rightarrow \ \lambda=3$

$\therefore \quad x^2 + y^2 + 3x = 0$

$$\Rightarrow \ x^2+y^2-y+3x+\left(\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2=\left(\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \ \left(x+\frac{3}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{\sqrt{10}}{2}\right)^2$$

3. (a, b, c, d) : Given, $l + m + n = 0$ ...(i)

and $2lm - mn + 2ln = 0$ ...(ii)

On solving (i) and (ii), we get

$$(l_1,m_1,n_1)=\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}}\right)$$

and $(l_2,m_2,n_2)=\left(\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$

Now, $\frac{l_1}{l_2}+\frac{m_1}{m_2}+\frac{n_1}{n_2}=1-\frac{1}{2}-2=\frac{-3}{2}$

$l_1l_2+m_1m_2+n_1n_2=\frac{1}{6}-\frac{2}{6}-\frac{2}{6}=\frac{-1}{2}$

$l_1m_1n_1+l_2m_2n_2=\frac{-2}{6\sqrt{6}}-\frac{2}{6\sqrt{6}}=\frac{-\sqrt{6}}{9}$

and $(l_1+l_2)(m_1+m_2)(n_1+n_2)=\frac{2}{\sqrt{6}}\times\frac{-1}{\sqrt{6}}\times\frac{-1}{\sqrt{6}}$

$=\frac{2}{6\sqrt{6}}=\frac{\sqrt{6}}{18}$

4. (b) : The line $\frac{x}{p}+\frac{y}{q}=1$, where p is a prime and q is a positive integer passes through the point (4, 3).

$\Rightarrow \ \frac{4}{p}+\frac{3}{q}=1$

$\Rightarrow \ q=\frac{3p}{p-4}=3+\frac{12}{p-4}$

Since, q is a positive integer, $p > 4$ and $(p - 4)$ divides 12. only two such primes are possible $p = 5$ and $p = 7$.

5. (a, b, c) : Line is parallel to the planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$

i.e., line is perpendicular to the vectors $\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+\hat{k}$

$\therefore$ Line is parallel to the vector $(\hat{i}-\hat{j}+2\hat{k})\times(3\hat{i}+\hat{j}+\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -3\hat{i}+5\hat{j}+4\hat{k}$$

Also, line passes through (1, 2, 3).

∴ Equation of line is

$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-3\hat{i}+5\hat{j}+4\hat{k}), \lambda \neq 0$

For $\lambda = 1$, we get the point (–2, 7, 7)

For $\lambda = -1$, we get the point (4, –3, –1)

For $\lambda = 2$, we get the point (–5, 12, 11)

6. (d) : Let the point S be (h, k).

According to given condition

$\{(h+1)^2 + k^2\} + \{(h-2)^2 + k^2\} = 2\{(h-1)^2 + k^2\}$

$\Rightarrow$ $2h + 3 = 0$, which is a straight line parallel to y-axis.

7. (14) : The distance of (3, 0, 0) from the plane $3x - 2y - z = 0$ is $\dfrac{9}{\sqrt{9+4+1}} = \dfrac{9}{\sqrt{14}}$

8. (5) : Volume $= \dfrac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \dfrac{9}{6} = \dfrac{3}{2}$

9. (2) : Let $G(x, y)$ is the centroid of triangle with vertices $P\left(\sqrt{2}\cos\theta, \sin\theta\right)$, $S(1, 0)$, $S_1(-1, 0)$, then

$3x = \sqrt{2}\cos\theta,\ 3y = \sin\theta$

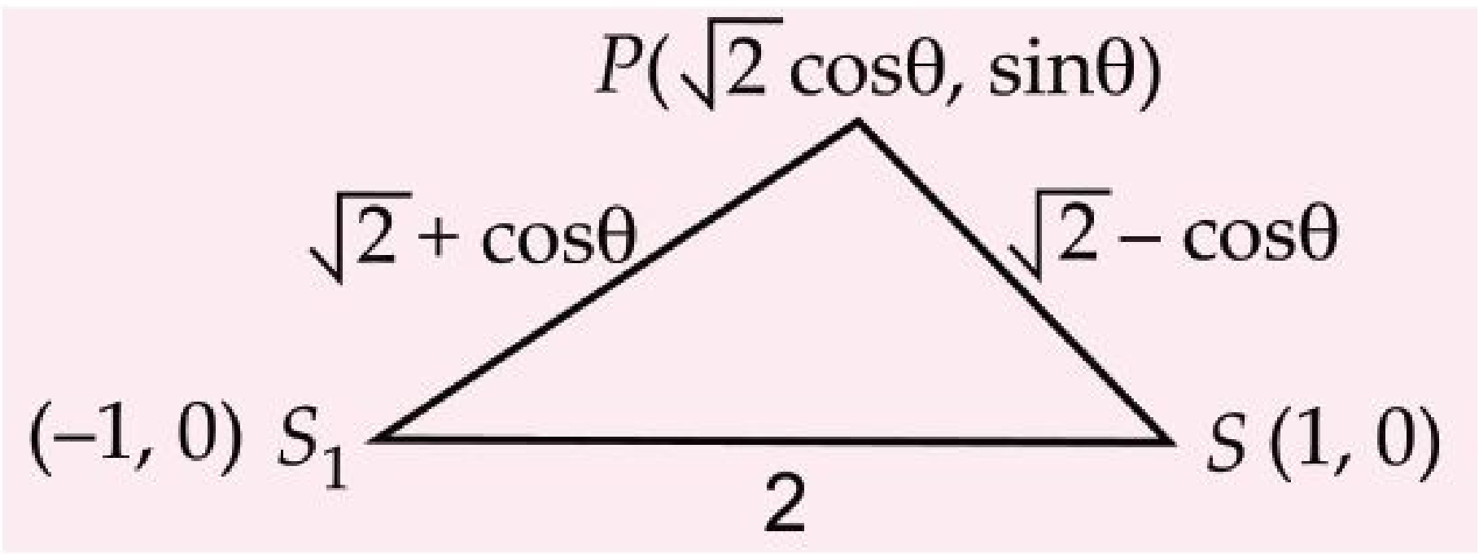
The locus of G is $\dfrac{x^2}{2} + y^2 = \dfrac{1}{9}$, eccentricity e is given by

$\dfrac{1}{9} = \dfrac{2}{9}(1-e^2) \Rightarrow e = \dfrac{1}{\sqrt{2}}$

10. (1) : $PS^2 = \left(\sqrt{2}\cos\theta - 1\right)^2 + \sin^2\theta = \left(\sqrt{2} - \cos\theta\right)^2$

$\therefore\ PS = \sqrt{2} - \cos\theta,\ PS_1 = \sqrt{2} + \cos\theta$

$SS_1 = 2$



If $I(x, y)$ is the incentre, then

$$x = \frac{2\sqrt{2}\cos\theta + \sqrt{2} + \cos\theta - \sqrt{2} + \cos\theta}{2 + \sqrt{2} - \cos\theta + \sqrt{2} + \cos\theta} = \cos\theta$$

$$y = \frac{2\sin\theta}{2 + 2\sqrt{2}} = \left(\sqrt{2} - 1\right)\sin\theta$$

The locus of I is $\dfrac{x^2}{1} + \dfrac{y^2}{\left(\sqrt{2}-1\right)^2} = 1$

Now, $\left(\sqrt{2}-1\right)^2 = 1 - e^2 \Rightarrow e = \sqrt{2\sqrt{2} - 2} = \sqrt{k\sqrt{k} - 2}$

$\therefore\ \dfrac{k}{2} = \dfrac{2}{2} = 1$

11. (5) : The vector perpendicular to L_1 and L_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}.$$

Required unit vector, $\hat{n} = \dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$.

12. (17) : The points on L_1 and L_2 are $A(-1, -2, -1)$ and $C(2, -2, 3)$

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The shortest distance

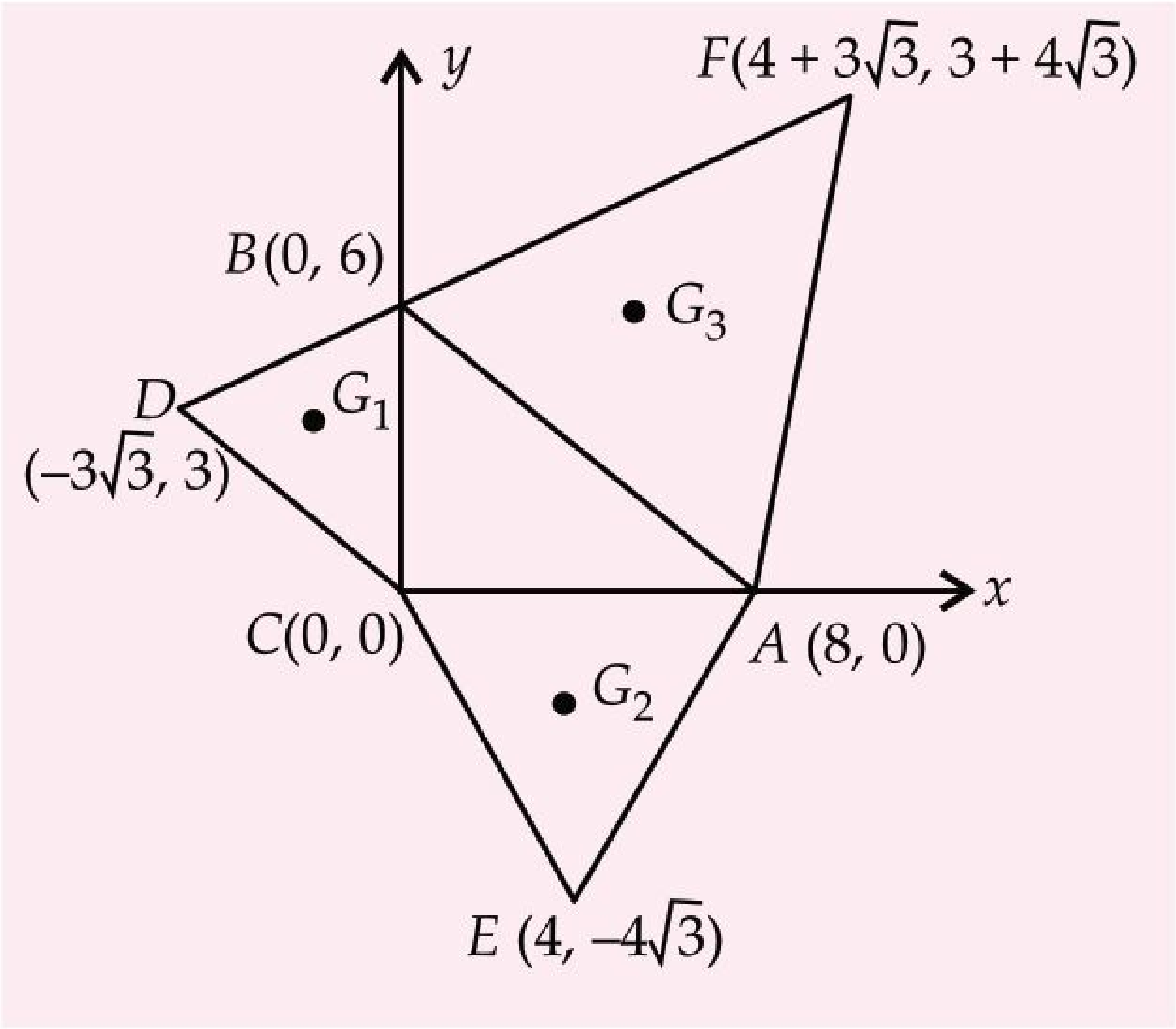
$$= \overrightarrow{AC} \cdot \hat{n} = (3\hat{i} + 4\hat{k}) \cdot \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

13. (d) : $D = (-3\sqrt{3}, 3)$, $E = (4, -4\sqrt{3})$,

$F = (4 + 3\sqrt{3}, 3 + 4\sqrt{3})$,

$$G_1 = (-\sqrt{3}, 3),\ G_2 = \left(4, \frac{-4}{\sqrt{3}}\right),\ G_3 = \left(4 + \sqrt{3}, 3 + \frac{4}{\sqrt{3}}\right)$$

$$x - y = 4 + 3\sqrt{3} - (3 + 4\sqrt{3}) = 1 - \sqrt{3}$$



14. (a) : $G_2 = \left(4, \frac{-4}{\sqrt{3}}\right), G_3 = \left(4 + \sqrt{3}, 3 + \frac{4}{\sqrt{3}}\right)$

$$(G_2 G_3)^2 = 3 + \left(3 + \frac{8}{\sqrt{3}}\right)^2 = 12 + \frac{64}{3} + 16\sqrt{3}$$

$$= \frac{100}{3} + 16\sqrt{3},\ G_2G_3 = \frac{2}{\sqrt{3}}\left(25 + 12\sqrt{3}\right)^{\frac{1}{2}}$$

15. (a) : $A = (2r, r, 1)$, $B = (2r, -r, -1)$ where $r \in R$.

Let $P = (\alpha, \beta, \gamma)$

For the first line,

$(2r - \alpha)^2 + (r - \beta)^2 + (1 - \gamma)^2$ is minimum

$$\Rightarrow \quad r = \frac{2\alpha + \beta}{5} \quad \therefore \quad A = \left(\frac{2}{5}(2\alpha + \beta), \frac{2\alpha + \beta}{5}, 1\right)$$

For the second line,

$(2r - \alpha)^2 + (r + \beta)^2 + (1 + \gamma)^2$ is minimum

$$\Rightarrow \quad r = \frac{2\alpha - \beta}{5}.$$

$$\therefore \quad B = \left(\frac{2}{5}(2\alpha - \beta), \frac{-1}{5}(2\alpha - \beta), -1\right)$$

$$OA \perp OB \Rightarrow \frac{4}{25}(4\alpha^2 - \beta^2) - \frac{1}{25}(4\alpha^2 - \beta^2) - 1 = 0$$

$\Rightarrow \quad 3(4\alpha^2 - \beta^2) = 25$

The locus is $3(4x^2 - y^2) = 25$.

16. (b) : $\because AP \perp BP$

$$\therefore \quad \left(\frac{2}{5}(2\alpha + \beta) - \alpha\right)\left(\frac{2}{5}(2\alpha - \beta) - \alpha\right) +$$

$$\left(\frac{2\alpha + \beta}{5} - \beta\right)\left(-\frac{1}{5}(2\alpha - \beta) - \beta\right) + (1 - \gamma)(-1 - \gamma) = 0$$

$\Rightarrow \quad 3(\alpha^2 - 4\beta^2) = 25\ (\gamma^2 - 1)$.

The locus is $3(x^2 - 4y^2) = 25\ (z^2 - 1)$.

17. (2) : As origin is shifted to (2, –3)

$\therefore \quad x \rightarrow x + 2,\ y \rightarrow y - 3$ and the equation is transformed to $(y - 3)^2 + 8(x + 2) + 6(y - 3) - 7 = 0$

$\Rightarrow \quad y^2 + 8x + 9 - 6y + 16 + 6y - 18 - 7 = 0$

$\Rightarrow \quad y^2 = -8x = 4(-2)x = 4(a)x \quad \therefore \quad a = -2$

18. (6) : The given equations can be written in the standard form as $\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2}$ and

$$\frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

The direction ratios of the lines are $-3, \frac{2p}{7}, 2$ and $\frac{-3p}{7}, 1, -5$ respectively.

It is given that the lines are perpendicular to each other.

$$\therefore \quad (-3) \cdot \left(\frac{-3p}{7}\right) + \left(\frac{2p}{7}\right) \cdot (1) + 2 \cdot (-5) = 0$$

$$\Rightarrow \quad \frac{9p}{7} + \frac{2p}{7} = 10 \Rightarrow 11p = 70 \Rightarrow p = \frac{70}{11} = 6.36$$

19. (13) : The equations of asymptotes are

$3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$

The combined equation of asymptotes is

$(3x - 4y + 7)(4x + 3y + 1) = 0$

or $\quad 12x^2 - 7xy - 12y^2 + 31x + 17y + 7 = 0 \quad$ ...(i)

Let the equation of hyperbola be

$12x^2 - 7xy - 12y^2 + 31x + 17y + A = 0 \quad$ ...(ii)

Since, (i) passes through (0, 1)

$\therefore \quad A = -5$

$\therefore \quad$ Equation of hyperbola is

$12x^2 - 7xy - 12y^2 + 31x + 17y - 5 = 0$

Equation of conjugate hyperbola

= 2(combined equation of asymptotes)

– (equation of hyperbola)

$\Rightarrow \quad$ Equation of conjugate hyperbola

$= 2(12x^2 - 7xy - 12y^2 + 31x + 17y + 7)$

$- (12x^2 - 7xy - 12y^2 + 31x + 17y - 5) = 0$

$\Rightarrow \quad 12x^2 - 7xy - 12y^2 + 31x + 17y + 19 = 0 \quad$ ...(iii)

Comparing (iii) with $12x^2 - 7xy - 12y^2 + ax + by + c = 0$, we have $a = 31$, $b = 17$, $c = 19$

$\Rightarrow \quad a + b + c = 31 + 17 + 19 = 67$

$\therefore \quad$ Required sum = 6 + 7 = 13

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. If $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$, then $2i\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$ equals

(a) $z_1\bar{z}_2 - z_2\bar{z}_1$ (b) $\bar{z}_1 z_2 - z_1\bar{z}_2$
(c) $|z_1|^2 - |z_2|^2$ (d) $|z_1|^2 - |z_1 - z_2|^2$

2. Let a, b, c be three distinct positive numbers which are in G.P. If $\log_c a$, $\log_b c$, $\log_a b$ are in A.P., then the common difference of the A.P. is

(a) 1/2 (b) 3/2 (c) 5/2 (d) 7/2

3. If $\cos^2 x - 2\cos x = 4\sin x - \sin 2x$, then

(a) $\tan(x/2) = -1/2$ (b) $\tan x = -1/2$
(c) $\tan(x/2) = 2 + \sqrt{5}$ (d) $\tan(x/2) = 2 - \sqrt{5}$

4. If a tower subtends angles θ, 2θ and 3θ at three points A, B and C respectively, lying on the same side of a horizontal line through the foot of the tower, AB/BC is equal to

(a) $\dfrac{\sin 2\theta}{\sin\theta}$ (b) $\dfrac{\sin 3\theta}{\sin 2\theta}$
(c) $\dfrac{\sin 3\theta}{\sin\theta}$ (d) $\dfrac{\cot\theta - \cot 2\theta}{\cot 2\theta - \cot 3\theta}$

5. If the line joining the points $A(a^2, 1)$ and $B(b^2, 1)$ is divided in the ratio $b : a$ at the point P whose x-coordinate is 7, then

(a) $ab = 7$ (b) $a^2 + b^2 = 7$
(c) $a^3 + b^3 = 14$ (d) $a^2 - ab + b^2 = 7$

6. Two points $(a, 3)$ and $(5, b)$ are the opposite vertices of a rectangle. If the coordinates (x, y) of the other two vertices satisfy the relation $y = 2x + c$, where $c^2 + 2a - b = 0$, then the value c can be

(a) $2\sqrt{2}+1$ (b) $2\sqrt{2}-1$
(c) $1-2\sqrt{2}$ (d) $-1-2\sqrt{2}$

7. The lengths of the perpendicular from the points $(m^2, 2m)$, $(mm', m + m')$ and $(m'^2, 2m')$ to the line $x + y + 1 = 0$ form

(a) an A.P. (b) a G.P.
(c) a H.P. (d) None of these

8. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. If two of its vertices are $(-3, 1)$ and $(1, 1)$, then the equation of the other side can be

(a) $7x - 4y + 25 = 0$ (b) $4x + 7y - 11 = 0$
(c) $4x - 7y + 5 = 0$ (d) $7x - 4y - 3 = 0$

9. α, β, γ are the parametric angles of three points P, Q and R respectively, on the circle $x^2 + y^2 = 1$ and A is the point $(-1, 0)$. If α, β, γ are in A.P. and AP, AQ, AR are in G.P., then

(a) $\cos(\alpha/2)$, $\cos(\beta/2)$, $\cos(\gamma/2)$ are in A.P.
(b) $\cos(\alpha/2)$, $\cos(\beta/2)$, $\cos(\gamma/2)$ are in G.P.
(c) $\sin(\alpha/2)$, $\sin(\beta/2)$, $\sin(\gamma/2)$ are in A.P.
(d) $\sin(\alpha/2)$, $\sin(\beta/2)$, $\sin(\gamma/2)$ are in G.P.

10. An ellipse passes through the point $(4, -1)$ and its axes are along the axes of coordinates. If the line $x + 4y - 10 = 0$ is a tangent to it, then its equation is

(a) $\dfrac{x^2}{100}+\dfrac{y^2}{5}=1$ (b) $\dfrac{x^2}{80}+\dfrac{y^2}{5/4}=1$
(c) $\dfrac{x^2}{20}+\dfrac{y^2}{5}=1$ (d) None of these

11. Which of the following statements are incorrect?

I. If $f(x)$ and $g(x)$ are one-one, then $f(x) + g(x)$ is also one-one.
II. If $f(x)$ and $g(x)$ are one-one, then $f(x)\cdot g(x)$ is also one-one.
III. If $f(x)$ is odd, then it is necessarily one-one.

(a) I and II only (b) II and III only
(c) III and I only (d) I, II and III

12. For $n \in N$, if

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{n}\right)=\frac{\pi}{4}$$

then n is equal to

(a) 43 (b) 47 (c) 49 (d) 51

13. If a non zero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$, then the leading coefficient of $f(x)$ is

(a) 1/6 (b) 1/9 (c) 1/12 (d) 1/18

14. If the sixth term in the expansion of

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}}+\frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$$

is 84, then $x =$

(a) 0 (b) 1 (c) 2 (d) 3

15. The $\lim\limits_{x\to 0} x^8\left[\dfrac{1}{x^3}\right]$ (where $[x]$ is greatest integer function) is

(a) a non-zero real number
(b) a rational number
(c) an integer
(d) zero

16. The slope of a tangent drawn from the point (1, 1) to the parabola $y^2 = 4(x - y)$ is

(a) 1 (b) – 1 (c) 1/2 (d) –1/2

17. If $2n$ boys are randomly divided into 2 teams containing n boys each, the probability that the 2 tallest boys are in different teams, is

(a) $\dfrac{n}{2n-1}$ (b) $\dfrac{n+1}{2n-1}$ (c) $\dfrac{n-1}{2n-1}$ (d) $\dfrac{n+1}{2n+1}$

18. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and l, m, n are distinct scalars, then $[(l\vec{a}+m\vec{b}+n\vec{c})(l\vec{b}+m\vec{c}+n\vec{a})(l\vec{c}+m\vec{a}+n\vec{b})]=0$ implies

(a) $lm + mn + nl = 0$ (b) $l + m + n = 0$
(c) $l^2 + m^2 + n^2 = 0$ (d) $l^3 + m^3 + n^3 = 0$

19. If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equation $x^2 + k = 0$, then

(a) $a + d = 0$ (b) $k = -|A|$
(c) $k = |A|$ (d) None of these

20. If $4l^2 - 5m^2 + 6l + 1 = 0$, then the line $lx + my + 1 = 0$ touches a fixed circle. Then the equation of circle is

(a) $(x + 3)^2 + (y - 0)^2 = 5$
(b) $(x - 3)^2 + (y - 0)^2 = 5$
(c) $(x - 3)^2 + (y - 0)^2 = 0$
(d) None of these

Comprehension Type

Paragraph for Q. No. 21 to 23

A bag contains 8 black, 3 red and 9 white balls. 3 balls are drawn at random.

21. The probability that there are 2 black balls and 1 white ball, is

(a) $\dfrac{21}{95}$ (b) $\dfrac{17}{95}$ (c) $\dfrac{23}{95}$ (d) $\dfrac{19}{96}$

22. The probability that the balls are one of each colour is

(a) $\dfrac{21}{95}$ (b) $\dfrac{18}{95}$ (c) $\dfrac{17}{95}$ (d) $\dfrac{23}{96}$

23. The probability that balls are drawn in the order black, red, white is

(a) $\dfrac{18}{95}$ (b) $\dfrac{21}{95}$ (c) $\dfrac{3}{95}$ (d) $\dfrac{7}{95}$

Paragraph for Q. No. 24 and 25

Let $\dfrac{dy}{dx} = \dfrac{(x+1)^2+y-3}{x+1}$, $y(1) = 1$.

24. The area bounded by the normal to the curve $y = y(x)$ at $x = 1$ and the coordinate axes is

(a) 4 (b) 3 (c) 2 (d) 1

25. The area bounded by the curve $y = y(x)$ and the lines $x = y$ and $x = 0$ is

(a) 1 (b) 1/2 (c) 1/3 (d) 2/3

Paragraph for Q. No. 26 to 28

Let $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$, and

$g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$.

26. Coefficient of x in $f(x)$ is

(a) $\dfrac{g(a)-f(b)}{b-a}$ (b) $\dfrac{g(-a)-g(-b)}{b-a}$
(c) $\dfrac{g(a)-g(b)}{b-a}$ (d) None of these

27. Which of the following is not a constant term in $f(x)$?

(a) $\dfrac{bg(a)-ag(b)}{(b-a)}$ (b) $\dfrac{bg(a)-af(-b)}{(b-a)}$
(c) $\dfrac{bf(-a)-ag(b)}{(b-a)}$ (d) None of these

28. Which of the following option is not true?
(a) $f(-a) = g(a)$
(b) $f(-a) = g(-a)$
(c) $f(a) = g(-a)$
(d) None of these

Matrix Match Type

29. Match the following :

Column-I		Column-II	
(A)	The remainder when 17^{20} is divided by 81, is	(P)	60
(B)	If $f(n) = \sum_{s=1}^{n}\sum_{r=s}^{n} {}^{n}C_r\, {}^{r}C_s$, then $f(3) =$	(Q)	19
(C)	The coefficient of x^2y in the expansion of $(1 + x + 2y)^5$ is	(R)	55
(D)	If $C_r = \binom{10}{r}$, then $\sum_{r=1}^{10} \frac{r \cdot C_r}{C_{r-1}}$ is equal to	(S)	46

30. Match the following.

Column-I		Column-II	
(A)	If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, where $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 2, \lvert\vec{c}\rvert = 1$, then the value of $[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]$ is	(P)	–12
(B)	If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at $\frac{\pi}{3}$, then the value of $16[\vec{a}\ \vec{b}+\vec{a}\times\vec{b}\ \vec{b}]$ is	(Q)	0
(C)	If $\vec{b}$ and $\vec{c}$ are orthogonal unit vectors and $\vec{b}\times\vec{c} = \vec{a}$, then the value of $[\vec{a}+\vec{b}+\vec{c}\ \vec{a}+\vec{b}\ \vec{b}+\vec{c}]$ is	(R)	16
(D)	If $[\vec{x}\ \vec{y}\ \vec{a}] = [\vec{x}\ \vec{y}\ \vec{b}] = [\vec{a}\ \vec{b}\ \vec{c}] = 0$, each vector being a non-zero vectors, then the value of $[\vec{x}\ \vec{y}\ \vec{c}]$ is	(S)	1

31. Match the following :

Column-I		Column-II	
(A)	If $y + \sin^2 x = xe^{xy}$, then $y'(0) =$	(P)	1
(B)	If $x + \lvert y\rvert = 2y$, then $y'(-1) + y'(1) =$	(Q)	$\frac{2}{3}$
(C)	If $f(x) = \sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}$ then $f'(3) + f'(6) =$	(R)	$\frac{4}{3}$
(D)	If $f(x) = \lvert\sin x - \cos x\rvert$, then $f'\left(\frac{\pi}{2}\right) =$	(S)	$\frac{1}{2}$

Numerical Answer Type

32. If $|z| = r$ and z_1, z_2 are the roots of the equation $z^2 + |z|z + \bar{z}z = 0$, then $|z_1 - z_2|^2 = \lambda r^2$, where λ is equal to ________.

33. The least positive angle measured in degrees satisfying the equation $\sin^3 x + \sin^3 2x + \sin^3 3x = (\sin x + \sin 2x + \sin 3x)^3$ is k°, then $\left[\frac{k}{19}\right]$ = ________.
(where $[\cdot]$ represents greatest integer function)

34. If the differential equation corresponding to the family of curves, $y = (A + Bx)e^{3x}$ is given by $\frac{d^2y}{dx^2} = a\frac{dy}{dx} + by$, then $(a - b)$ equals ________.

35. If $[x^2] + x = a$ has solution(s) $\forall\ a \in N$, then number of values of 'a' less than 20 is ________.
(where $[\cdot]$ denotes greatest integer function)

36. Number of non negative integer values of x in the domain of $y(x)$ given by equation $3^x + 2^y = 250$ is ________.

37. The number of solutions of the equation

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 0 \text{ in } (-\pi, \pi)$$

is ________.

38. Let k be a positive real number and

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6, then $[k]$ is equal to ________.

39. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is ________.

40. A biquadratic function $y = ax^4 + bx^3 + cx^2 + dx + e$ $(a \neq 0)$ touches $y = px + q$ at $x = \alpha, \beta$ $(\alpha < \beta)$. Area of the region bounded by these graphs is $\frac{a(\beta-\alpha)^5}{\lambda}$ then $\frac{\lambda}{5}$ = ________.

SOLUTIONS

1. (b): $2i\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = 2i(x_1y_2 - y_1x_2)$

$= x_1x_2 + ix_1y_2 - iy_1x_2 - i^2y_1y_2 - (x_1x_2 - ix_1y_2 + (iy_1x_2 - i^2y_1y_2)$

$= x_1(x_2 + iy_2) - iy_1(x_2 + iy_2) - [x_1(x_2 - iy_2) + iy_1(x_2 - iy_2)]$

$= (x_1 - iy_1)(x_2 + iy_2) - (x_2 - iy_2)(x_1 + iy_1)$

$= \bar{z}_1 z_2 - z_1 \bar{z}_2$

2. (b): Let $x = \log_c a$, $y = \log_b c$, $z = \log_a b$

We have $xyz = 1$,

Also, $2y = x + z$ and $b^2 = ac$

$\Rightarrow \quad 2\log b = \log a + \log c \Rightarrow 2 = \frac{1}{z} + y \Rightarrow 2z = 1 + yz$

$\Rightarrow \quad 2xz = x + xyz = x + 1$

Thus, $y = 2 - 1/z$, $x = 1/(2z - 1)$

Now, $2y = x + z$

$\Rightarrow \; 4 - \frac{2}{z} = \frac{1}{2z-1} + z \Rightarrow \quad 2z^3 - 9z^2 + 9z - 2 = 0$

As $z \neq 1$, $2z^2 - 7z + 2 = 0$

Now, $d = z - y = z + \frac{1}{z} - 2$

$= \frac{z^2 - 2z + 1}{z} = \frac{7z/2 - 2z}{z} = \frac{3}{2}$

3. (a,c,d) : We have,

$\cos^2 x - 2\cos x = 4\sin x - 2\sin x\cos x$

$\Rightarrow \quad (\cos x - 2)(\cos x + 2\sin x) = 0 \Rightarrow \tan x = -1/2$

$\Rightarrow \quad -1/2 = \frac{2\tan(x/2)}{1 - \tan^2(x/2)}$

$\Rightarrow \quad \tan^2(x/2) - 4\tan(x/2) - 1 = 0$

$\Rightarrow \quad \tan(x/2) = 2 \pm \sqrt{5}$

4. (c,d) : Let h metres be the height of tower PQ.

Now, $AB = AQ - BQ = h\cot\theta - h\cot 2\theta$ ...(i)

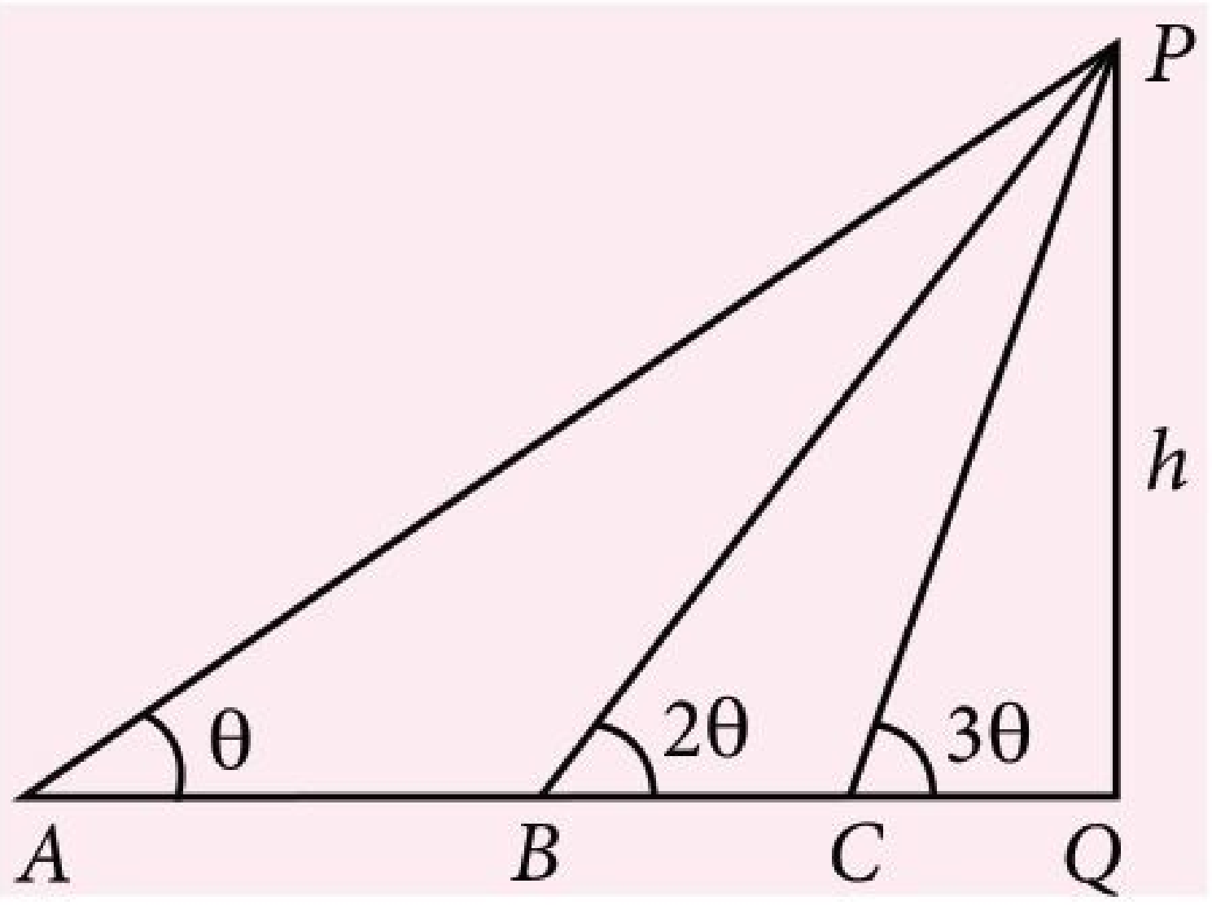
Also, $BC = BQ - CQ$

$= h\cot 2\theta - h\cot 3\theta$ ...(ii)

From (i) and (ii),

$\frac{AB}{BC} = \frac{\cot\theta - \cot 2\theta}{\cot 2\theta - \cot 3\theta}$

$= \frac{\sin 3\theta}{\sin\theta}$



5. (d): Using section formula, we have,

$\frac{b \cdot b^2 + a \cdot a^2}{a+b} = 7 \Rightarrow \frac{b^3 + a^3}{a+b} = 7 \Rightarrow a^2 - ab + b^2 = 7$

6.(a, c): Mid-point of the other vertices is also the mid-point of the given vertices and hence satisfies the given relation.

So, $\frac{b+3}{2} = 2\left(\frac{a+5}{2}\right) + c \Rightarrow 2a + 2c - b + 7 = 0$

Also $c^2 + 2a - b = 0 \Rightarrow c^2 - 2c - 7 = 0 \Rightarrow c = 1 \pm 2\sqrt{2}$

7. (b): $p_1 = \frac{m^2 + 2m + 1}{\sqrt{2}}$, $p_2 = \frac{mm' + m + m' + 1}{\sqrt{2}}$,

$p_3 = \frac{m'^2 + 2m' + 1}{\sqrt{2}}$

$\Rightarrow \quad p_1p_3 = p_2^2 \Rightarrow p_1, p_2, p_3$ are in G.P.

8. (a,b,d) : $(-3, 1)$ lies on the given line.

Equation of the line passes through the point $(-3, 1)$ and perpendicular to the given line is $7x - 4y + 25 = 0$. Others two sides passes through the point $(1, 1)$ and are parallel to these two sides.

9. (b,d) : $P(\cos\alpha, \sin\alpha)$, $Q(\cos\beta, \sin\beta)$, $R(\cos\gamma, \sin\gamma)$

$\therefore AP = 2\cos(\alpha/2)$, $AQ = 2\cos(\beta/2)$, $AR = 2\cos(\gamma/2)$

Since, AP, AQ and AR are in G.P.

$\cos(\alpha/2)$, $\cos(\beta/2)$, $\cos(\gamma/2)$ are in G.P.

$\Rightarrow \quad \cos(\alpha/2)\cos(\gamma/2) = \cos^2(\beta/2)$

Since α, β, γ are in A.P. $\quad \therefore \frac{\alpha+\gamma}{2} = \beta$

$\Rightarrow \quad \cos(\alpha/2)\cos(\gamma/2) - \sin(\alpha/2)\sin(\gamma/2) = \cos\beta$

$\Rightarrow \quad \cos^2(\beta/2) - \cos\beta = \sin(\alpha/2)\sin(\gamma/2)$

$\Rightarrow \quad 1 - \cos^2(\beta/2) = \sin(\alpha/2)\sin(\gamma/2)$

$\Rightarrow \quad \sin^2(\beta/2) = \sin(\alpha/2)\sin(\gamma/2)$

$\Rightarrow \quad \sin(\alpha/2)$, $\sin(\beta/2)$, $\sin(\gamma/2)$ are in G.P.

10. (b,c) : Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

It passes through $(4, -1)$

$\therefore \; \frac{16}{a^2} + \frac{1}{b^2} = 1 \Rightarrow a^2 + 16b^2 = a^2b^2$

Also, line $x + 4y - 10 = 0$ touches the ellipse.

$\left(\frac{10}{4}\right)^2 = a^2\left(-\frac{1}{4}\right)^2 + b^2 \Rightarrow a^2 + 16b^2 = 100$

$\Rightarrow \quad a^2b^2 = 100$

$\therefore$ Required equations can be of option (b) and (c).

11. (d): I. Let $f(x) = x$ and $g(x) = -x$

Here, both $f(x)$ and $g(x)$ are one-one but $f(x) + g(x) = 0$ is not one-one.

II. Let $f(x) = x$ and $g(x) = x^3$

Here $f(x)$ and $g(x)$ are both one-one but $f(x)\cdot g(x) = x^4$ is not one-one.

III. $f(x) = \sin x$, which is odd but not one-one

$\therefore$ All the three statements are incorrect.

12. (b): We have, $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}=\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{12}}\right)$

$$=\tan^{-1}\left(\frac{7}{11}\right)$$

Again, $\tan^{-1}\frac{7}{11}+\tan^{-1}\frac{1}{5}=\tan^{-1}\left(\frac{\frac{7}{11}+\frac{1}{5}}{1-\frac{7}{55}}\right)$

$$=\tan^{-1}\left(\frac{46}{48}\right)=\tan^{-1}\frac{23}{24}$$

$$\therefore\ \tan^{-1}\left(\frac{1}{n}\right)=\tan^{-1}(1)-\tan^{-1}\left(\frac{23}{24}\right)=\tan^{-1}\left(\frac{1-\frac{23}{24}}{1+\frac{23}{24}}\right)$$

$$=\tan^{-1}\left(\frac{1}{47}\right)\ \Rightarrow\ n=47$$

13. (d): Degree of $f(x) = n$; Degree of $f'(x) = (n - 1)$

Degree of $f''(x) = (n - 2)$

Hence, $n = (n - 1) + (n - 2) = 2n - 3 \quad \therefore\ n = 3$

Hence, $f(x) = ax^3 + bx^2 + cx + d, (a \neq 0)$

$f'(x) = 3ax^2 + 2bx + c;\ \ f''(x) = 6ax + 2b$

$\therefore\quad ax^3 + bx^2 + cx + d = (3ax^2 + 2bx + c)\,(6ax + 2b)$

$\therefore\quad 18a^2 = a \Rightarrow a = 1/18$

14. (b, c) : $2^{\log_2\sqrt{9^{x-1}+7}}=\sqrt{9^{x-1}+7}$

and $2^{\frac{1}{5}\log_2(3^{x-1}+1)}=(3^{x-1}+1)^{1/5}$

Now, $T_6=\binom{7}{5}\left(\sqrt{9^{x-1}+7}\right)^2(3^{x-1}+1)^{-1}=84$

$$\Rightarrow\ \frac{9^{x-1}+7}{3^{x-1}+1}=\frac{84}{21}=4$$

$\Rightarrow\quad (3^{x-1})^2 - 4(3^{x-1}) + 3 = 0$

$\Rightarrow\quad (3^{x-1} - 1)\,(3^{x-1} - 3) = 0$

$\Rightarrow\quad 3^{x-1} = 1, 3 \Rightarrow x = 1, 2$

15. (b, c, d) : Since $x - 1 \le [x] \le x$ for all $x \in R$ so

$$\Rightarrow\ \frac{1}{x^3}-1\le\left[\frac{1}{x^3}\right]\le\frac{1}{x^3}$$

$$\Rightarrow x^8\left(\frac{1}{x^3}-1\right)\le x^8\left[\frac{1}{x^3}\right]\le x^5\quad\text{for all } x$$

$$\Rightarrow\ x^5-x^8\le x^8\left[\frac{1}{x^3}\right]\le x^5$$

But $\lim\limits_{x\to0}x^5=0=\lim\limits_{x\to0}\left(x^5-x^8\right)$

So, $\lim\limits_{x\to0}x^8\left[\frac{1}{x^3}\right]=0\in I\subseteq Q.$

16. (a, c) : Let m be the slope of the tangent from (1, 1). The line $y - 1 = m\,(x - 1)$ meets the parabola at two coincident points.

Eliminating y between the line and the parabola,

$(1 + m\,(x - 1))^2 - 4x + 4\,(1 + m\,(x - 1)) = 0$

$\Rightarrow\quad m^2\,(x - 1)^2 + (6m - 4)\,(x - 1) + 1 = 0$

$\therefore\quad (6m - 4)^2 - 4m^2 = 0$

$\Rightarrow\quad (6m - 4 - 2m)(6m - 4 + 2m) = 0$

$\Rightarrow\quad (m - 1)\,(8m - 4) = 0 \Rightarrow m = 1, \frac{1}{2}$

17. (a): The number of ways of dividing $2n$ boys into 2 teams is $\binom{2n-2}{n-1}\frac{1}{2!}$

(Here, we exclude the two tallest boys say A and B and form two teams in ways).

$$\therefore\ \text{Required probability}=\binom{2n-2}{n-1}\cdot\frac{1}{2!}\times2\div\left(\binom{2n}{n}\frac{1}{2!}\right)$$

$$=\frac{2n^2}{2n(2n-1)}=\frac{n}{2n-1}$$

18. (b): $\left.\begin{aligned}\vec{V}_1&=l\vec{a}+m\vec{b}+n\vec{c}\\ \vec{V}_2&=n\vec{a}+l\vec{b}+m\vec{c}\\ \vec{V}_3&=m\vec{a}+n\vec{b}+l\vec{c}\end{aligned}\right\},$

where $\vec{a},\vec{b},\vec{c}$ are non-coplanar.

$$\therefore\ [\vec{V}_1\,\vec{V}_2\,\vec{V}_3]=\begin{vmatrix} l & m & n\\ n & l & m\\ m & n & l\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]=0$$

But, $[\vec{a}\,\vec{b}\,\vec{c}]\neq0$

$$\Rightarrow\quad\begin{vmatrix} l & m & n\\ n & l & m\\ m & n & l\end{vmatrix}=0$$

$\Rightarrow\quad l(l^2 - mn) - m(ln - m^2) + n(n^2 - lm) = 0$

$\Rightarrow\quad l^3 + m^3 + n^3 - 3lmn = 0$

$\Rightarrow\quad (l + m + n)\,(l^2 + m^2 + n^2 - lm - mn - ln) = 0$

$\Rightarrow\quad l + m + n = 0$

19. (a, c) : We have,

$$A^2=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a^2+bc & ab+db\\ ac+cd & bc+d^2\end{bmatrix}$$

As, A satisfies, $x^2 + k = 0$, $A^2 + kI = O$

$$\Rightarrow \begin{bmatrix} a^2+bc+k & (a+d)b \\ (a+d)c & bc+d^2+k \end{bmatrix} = O$$

$\Rightarrow \quad a^2+bc+k=0\ ;\ bc+d^2+k=0$

and $(a+d)b=(a+d)c=0$

As $bc\neq 0 \Rightarrow b\neq 0, c\neq 0 \Rightarrow a+d=0$

Also, $k=-(a^2+bc)=-(d^2+bc)=-[-ad+bc]=|A|$

20. (b): Let the line $lx+my+1=0$ touches the circle $(x-a)^2+(y-b)^2=r^2$

Then, $\left|\dfrac{la+mb+1}{\sqrt{l^2+m^2}}\right|=r$

$\Rightarrow \quad (la+mb+1)^2=r^2(l^2+m^2)$

$\Rightarrow l^2(a^2-r^2)+m^2(b^2-r^2)+2ablm+2al+2bm+1=0$

On comparing this with the equation,

$4l^2-5m^2+6l+1=0$, we have

$a^2-r^2=4,\ b^2-r^2=-5,\ 2ab=0,\ 2a=6,\ 2b=0$

$\Rightarrow \quad a=3,\ b=0$ and $r^2=5$

Thus, the line touches the fixed circle

$(x-3)^2+(y-0)^2=5$.

21. (a): Required probability

$$=\frac{\binom{8}{2}\binom{9}{1}}{\binom{20}{3}}=\frac{8\cdot7\cdot9\cdot3}{20\cdot19\cdot18}=\frac{21}{95}.$$

22. (b): Required probability

$$=\frac{\binom{8}{1}\binom{3}{1}\binom{9}{1}}{\binom{20}{3}}=\frac{8\cdot3\cdot9\cdot6}{20\cdot19\cdot18}=\frac{18}{95}$$

23. (c): Required probability $=\dfrac{8}{20}\cdot\dfrac{3}{19}\cdot\dfrac{9}{18}=\dfrac{3}{95}$

24. (c): Putting $x+1=X,\ y-3=Y$, we get

$$\Rightarrow \quad \frac{dY}{dX}=X+\frac{Y}{X} \Rightarrow \frac{dY}{dX}-\frac{Y}{X}=X,$$

which is linear differential equation, with I.F. $=\dfrac{1}{X}$

$\therefore \quad$ Solution is given by

$$\frac{Y}{X}=\int X\cdot\frac{1}{X}dX=X+c$$

$\Rightarrow Y=X(X+c) \Rightarrow y=3+(x+1)(x+1+c)$

Now, $y(1)=1 \Rightarrow 1=3+2(2+c) \Rightarrow c=-3$

$\therefore \quad y=x^2-x+1,\ y'=2x-1$

$y'(1)=1$

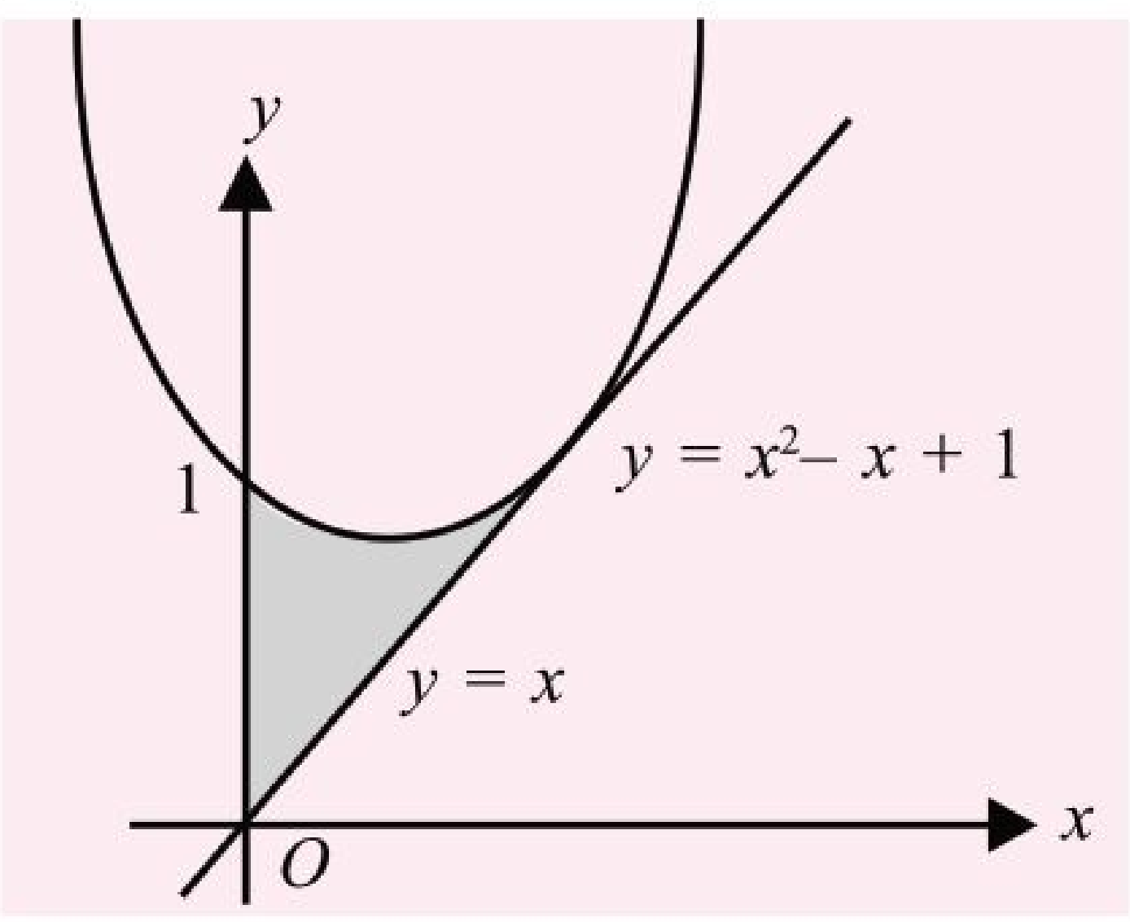
Normal at $(1, 1)$ is $y-1=-1(x-1)$

$\Rightarrow \quad x+y=2$, which meets x-axis at $A(2, 0)$ and y-axis at $B(0, 2)$.

$\therefore \quad$ Area of $\Delta OAB=\dfrac{1}{2}\cdot2\cdot2=2$

25. (c): $x=y$ is a tangent to the curve at $(1, 1)$.

$\therefore \quad$ Area $=\displaystyle\int_0^1(x^2-x+1-x)dx$



$$=\int_0^1(x-1)^2\,dx=\left[\frac{(x-1)^3}{3}\right]_0^1=\frac{1}{3}$$

(26-28) :

26. (c) **27. (d)** **28. (b)**

In given determinant applying $C_2\to C_2-C_1$ and $C_3\to C_3-C_2$, we get

$$f(x)=\begin{vmatrix} x+c_1 & a-c_1 & 0 \\ x+b & c_2-b & a-c_2 \\ x+b & 0 & c_3-b \end{vmatrix}$$

$$=x\begin{vmatrix} 1 & a-c_1 & 0 \\ 1 & c_2-b & a-c_2 \\ 1 & 0 & c_3-b \end{vmatrix}+\begin{vmatrix} c_1 & a-c_1 & 0 \\ b & c_2-b & a-c_2 \\ b & 0 & c_3-b \end{vmatrix}$$

So, $f(x)$ is linear.

Let $f(x)=Px+Q$, then $f(-a)=-aP+Q$,

$f(-b)=-bP+Q$

Then $f(0)=0\cdot P+Q=Q=\dfrac{bf(-a)-af(-b)}{b-a}$ ...(i)

Also $f(-a)=\begin{vmatrix} c_1-a & 0 & 0 \\ b-a & c_2-a & 0 \\ b-a & b-a & c_3-a \end{vmatrix}$

$=(c_1-a)(c_2-a)(c_3-a)$

Similarly $f(-b)=(c_1-b)(c_2-b)(c_3-b)$

And $g(x)=(c_1-x)(c_2-x)(c_3-x)$

$\Rightarrow \quad g(a)=f(-a)$ and $g(b)=f(-b)$

Now from (i), we get $f(0)=\dfrac{bg(a)-ag(b)}{b-a}$

Also $f(-a)-f(-b)=(b-a)P$

$\therefore \quad P=\dfrac{g(a)-g(b)}{b-a}$

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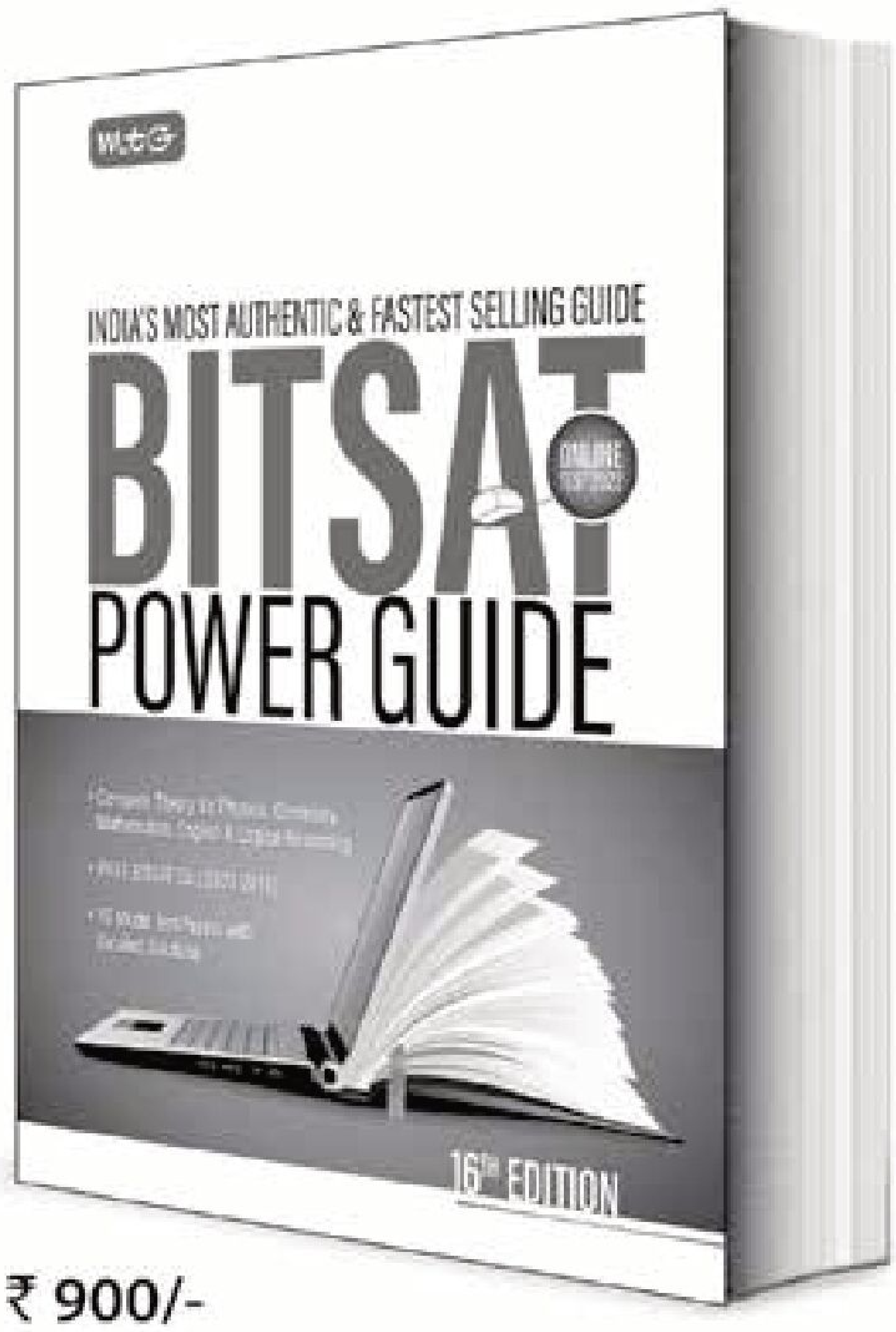

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29. (A) → (S), (B) → (Q), (C) → (P), (D) → (R)

(A) : $17^{20} = (18 - 1)^{20} = 1 - 20 \times 18 +$ multiple of 18^2
$= -35 +$ multiple of $9^2 = 46 +$ multiple of 81.

(B) : We have, $f(n) = \sum_{s=1}^{n}\sum_{r=s}^{n} {}^nC_r\, {}^rC_s$

$$\Rightarrow f(n) = \sum_{r=1}^{n} {}^nC_r\, {}^rC_1 + \sum_{r=2}^{n} {}^nC_r\, {}^rC_2 + \dots. + \sum_{r=n-1}^{n} {}^nC_r\, {}^rC_{n-1} + {}^nC_n \quad {}^nC_n$$

$$\Rightarrow f(n) = {}^nC_1\, {}^1C_1 + {}^nC_2\left({}^2C_1 + {}^2C_2\right) + {}^nC_3\left({}^3C_1 + {}^3C_2 + {}^3C_3\right) + \dots. + {}^nC_n\left({}^nC_1 + {}^nC_2 + \dots. + {}^nC_n\right)$$

$$\Rightarrow f(n) = {}^nC_1 \times \left(2^1 - 1\right) + {}^nC_2 \times \left(2^2 - 1\right) + {}^nC_3 \times \left(2^3 - 1\right) + \dots + {}^nC_n \times \left(2^n - 1\right)$$

$$\Rightarrow f(n) = \sum_{r=1}^{n} {}^nC_r\, 2^r - \sum_{r=1}^{n} {}^nC_r$$

$$\Rightarrow f(n) = \sum_{r=0}^{n} {}^nC_r\, 2^r - \sum_{r=0}^{n} {}^nC_r$$

$$\Rightarrow f(n) = 3^n - 2^n \Rightarrow f(3) = 3^3 - 2^3 = 19$$

(C) : $(1 + (x + 2y))^5 =$

$$= 1 + \binom{5}{1}(x + 2y) + \binom{5}{2}(x + 2y)^2 + \binom{5}{3}(x + 2y)^3 + \dots$$

The coefficient of x^2y is $\binom{5}{3} \cdot 3 \cdot 2 = 60$

(D) : $\sum_{r=1}^{10} r \cdot \frac{C_r}{C_{r-1}} = \sum_{r=1}^{10} r \cdot \frac{(10 - r + 1)}{r} = \sum_{r=1}^{10} (11 - r)$

$$= 10 + 9 + 8 + \dots + 2 + 1 = \frac{10 \cdot 11}{2} = 55$$

30. (A) → (R), (B) → (P), (C) → (S), (D) → (Q)

(A) If $\vec{a}, b, \vec{c}$ are mutually perpendicular,

$$[\vec{a} \times \vec{b}\ \vec{b} \times \vec{c}\ \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2 = (|\vec{a}||\vec{b}||\vec{c}|)^2 = 16$$

(B) Given $\vec{a}, \vec{b}$ are two unit vectors *i.e.*, $|\vec{a}| = |\vec{b}| = 1$ and angle between them is $\pi/3$.

$$\therefore \sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} \Rightarrow \sin\pi/3 = |\vec{a} \times \vec{b}| \Rightarrow \frac{\sqrt{3}}{2} = |\vec{a} \times \vec{b}|$$

Now, $[\vec{a}\ \ \vec{b} + \vec{a} \times \vec{b}\ \ \vec{b}] = [\vec{a}\ \vec{b}\ \vec{b}] + [\vec{a}\ \vec{a} \times \vec{b}\ \vec{b}]$

$$= 0 + [\vec{a}\ \vec{a} \times \vec{b}\ \vec{b}] = (\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{a})$$

$$= -(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b}) = -|\vec{a} \times \vec{b}|^2 = -\frac{3}{4}$$

(C) If $\vec{b}$ and $\vec{c}$ are orthogonal

$\therefore$ $\vec{b} \cdot \vec{c} = 0$ and also given $\vec{b} \times \vec{c} = \vec{a}$

Now $[\vec{a} + \vec{b} + \vec{c}\ \ \vec{a} + b\ \ b + \vec{c}]$

$$= [\vec{a}\ \ \vec{a} + b\ \ \vec{b} + \vec{c}] + [\vec{b} + \vec{c}\ \ \vec{a} + \vec{b}\ \ \vec{b} + \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$$

$= \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot \vec{a} = |\vec{a}|^2 = 1$ $\qquad$ [$\because \vec{a}$ is unit vector]

(D) $[\vec{x}\ \vec{y}\ \vec{a}] = 0$

Hence, $\vec{x}, \vec{y}$ and $\vec{a}$ are coplanar. ...(i)

Also, $[\vec{x}\ \vec{y}\ \vec{b}] = 0$

Hence, $\vec{x}, \vec{y}$ and $\vec{b}$ are coplanar ...(ii)

Also, $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

Hence, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar. ...(iii)

From (i) (ii) and (iii), we get
$\vec{x}, \vec{y}$ and $\vec{c}$ are coplanar. $\therefore [\vec{x}\ \vec{y}\ \vec{c}] = 0$

31. (A) → (P), (B) → (R), (C) → (S), (D) → (P)

(A) $y + \sin^2 x = xe^{xy} \Rightarrow y(0) = 0$

Also, $y' + 2\sin x \cos x = e^{xy} + xe^{xy}[xy' + y]$

$\Rightarrow$ $y'(0) = 1$

(B) $x + |y| = 2y$

$$\Rightarrow y = x, x \geq 0, y = \frac{x}{3}, x < 0$$

Also, $y'(-1) + y'(1) = \frac{1}{3} + 1 = \frac{4}{3}$

(C) $f(x) = \sqrt{x - 2} + \sqrt{2} + \left|\sqrt{x - 2} - \sqrt{2}\right|$

$\therefore$ $f(x) = 2\sqrt{x - 2}$ for $x \geq 4$ and $2\sqrt{2}$ for $x < 4$

$$\therefore f'(3) + f'(6) = 0 + \frac{1}{\sqrt{6 - 2}} = \frac{1}{2}$$

(D) If $f(x) = \sqrt{2}\left|\sin\left(x - \frac{\pi}{4}\right)\right| = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

$$\therefore f'(x) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) \Rightarrow f'\left(\frac{\pi}{2}\right) = 1$$

32. (3): $z = re^{i\theta} \Rightarrow e^{2i\theta} + e^{i\theta} + 1 = 0$

$\Rightarrow$ $\cos 2\theta + \cos\theta + 1 = 0, \sin 2\theta + \sin\theta = 0$

$$\Rightarrow \theta = \pm\frac{2\pi}{3}, z_1 = re^{\frac{i2\pi}{3}} = r\omega$$

$z_2 = re^{\frac{-2\pi i}{3}} = r\omega^2$, where ω is a complex cube root of unity

$|z_1 - z_2|^2 = r^2|\omega - \omega^2|^2$

$$= r^2\left|-\frac{1}{2} + \frac{i\sqrt{3}}{2} + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right|^2 = r^2|i\sqrt{3}|^2 = 3r^2$$

33. (3): $\because\ (a+b+c)^3$
$= a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$
$\therefore\ \sin^3 x + \sin^3 2x + \sin^3 3x - (\sin x + \sin 2x + \sin 3x)^3 = 0$
$\Rightarrow\ -3(\sin x + \sin 2x)(\sin 2x + \sin 3x)(\sin 3x + \sin x) = 0$

$$\therefore\quad \sin x + \sin 2x = 0 \Rightarrow 2\sin\frac{3x}{2}\cdot\cos\frac{x}{2} = 0$$

$$\Rightarrow\ \sin\frac{3x}{2} = \sin x \Rightarrow x = \frac{2\pi}{3} = 120^\circ$$

$$\text{and } \cos\frac{x}{2} = \cos\frac{\pi}{2} \Rightarrow x = \pi = 180^\circ$$

Now, $\sin 2x + \sin 3x = 0$

$$\Rightarrow 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0 \Rightarrow \sin\frac{5x}{2} = \sin\pi$$

$$\Rightarrow\ x = \frac{2\pi}{5} = 72^\circ$$

$$\text{and } \cos\frac{x}{2} = \cos\frac{\pi}{2} \Rightarrow x = \pi = 180^\circ$$

Also, $\sin 3x + \sin x = 0 \Rightarrow 2\sin 2x \cos x = 0$

$$\Rightarrow \sin 2x = \sin\pi \Rightarrow x = \frac{\pi}{2} = 90^\circ.$$

$$\text{and } \cos x = \cos\frac{\pi}{2} \Rightarrow x = \frac{\pi}{2} = 90^\circ$$

$\therefore$ Least value of x is 72°

$$\text{Hence, } \left[\frac{k}{19}\right] = \left[\frac{72^\circ}{19}\right] = 3$$

34. (15): Given equation can be written as
$y\cdot e^{-3x} = A + Bx$
$\Rightarrow\ e^{-3x}\cdot y_1 - 3y\cdot e^{-3x} = B$
$\Rightarrow\ e^{-3x}\cdot y_2 - 3y_1\cdot e^{-3x} - 3[e^{-3x}\cdot y_1 - 3y\cdot e^{-3x}] = 0$
$\Rightarrow\ y_2 - 3y_1 - 3y_1 + 9y = 0$

$$\Rightarrow\ \frac{d^2y}{dx^2} = 6\frac{dy}{dx} - 9y \Rightarrow a = 6 \text{ and } b = -9$$

$\therefore\ a - b = 6 + 9 = 15$

35. (3): $x = a - [x^2]$
$\Rightarrow\ x = a - x^2 \Rightarrow a = x(x+1)$
$\therefore$ a can be 2, 6, 12, 20..... .
$\therefore$ Number of values of 'a' less than 20 is 3.

36. (6): $2^y = 250 - 3^x$
$\Rightarrow\ \log_2 2^y = \log_2(250 - 3^x)$
$\Rightarrow\ y = \log_2(250 - 3^x)$
Here, $250 - 3^x > 0 \Rightarrow x < \log_3 250$
$\therefore\ x \in (-\infty, \log_3 250)$
$\therefore$ Non-negative integral value = {0, 1, 2, 3, 4, 5}

37. (5): $t = \tan x \Rightarrow t + \dfrac{t+\sqrt{3}}{1-\sqrt{3}t} + \dfrac{t-\sqrt{3}}{1+\sqrt{3}t} = 0$

$\Rightarrow\ t(1-3t^2) + 8t = 0 \Rightarrow t^3 - 3t = 0$

$$\therefore\ t = 0, \pm\sqrt{3},\ x = 0, \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}.$$

38. (4): $\det A = (2k-1)(4k^2-1) + 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k}) + 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k})$
$= (2k-1)(4k^2-1) + 8(2k+1)k$
$= (2k+1)((2k-1)^2 + 8k) = (2k+1)^3$
$\therefore\ \det(\text{adj } A) = (\det A)^2 = (2k+1)^6$
$\det B = 0$ since B is skew symmetric matrix of order 3.
$\therefore\ \det(\text{adj } B) = (\det B)^2 = 0$
$\therefore\ (2k+1)^6 = 10^6$
$\Rightarrow\ 2k + 1 = 10 \Rightarrow k = 4.5$
So, $[k] = [4.5] = 4$

39. (2): $f(x) = x^3 + e^{x/2},\ g(x) = f^{-1}(x)$
Now, $f(g(x)) = x$ and $f(0) = 1$
$\Rightarrow\ f^{-1}(1) = 0 \Rightarrow g(1) = 0$
$\therefore\ f'(g(x))\cdot g'(x) = 1$

$$\Rightarrow\ g'(x) = \frac{1}{f'(g(x))}$$

$$\Rightarrow g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)}$$

$$\therefore\ f'(x) = 3x^2 + \frac{1}{2}e^{x/2} \Rightarrow f'(0) = \frac{1}{2}\quad \text{So, } g'(1) = 2.$$

40. (6): For $a > 0$, $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
Since line $g(x) = px + q$ touches $y = f(x)$ at two points $x = \alpha, \beta$
$\therefore\ |f(x) - g(x)| = a(x-\alpha)^2(x-\beta)^2$

$$\therefore\quad \text{Area} = \int_\alpha^\beta |f(x) - g(x)|\,dx = a\int_\alpha^\beta (x-\alpha)^2(x-\beta)^2\,dx$$

$$= a\left\{\left[(x-\alpha)^2\frac{(x-\beta)^3}{3}\right]_\alpha^\beta - \frac{2}{3}\int_\alpha^\beta (x-\alpha)(x-\beta)^3\,dx\right\}$$

$$= -\frac{2a}{3}\left\{\left[(x-\alpha)\frac{(x-\beta)^4}{4}\right]_\alpha^\beta - \frac{1}{4}\int_\alpha^\beta (x-\beta)^4\,dx\right\}$$

$$= \frac{a}{6}\int_\alpha^\beta (x-\beta)^4\,dx = \left[\frac{a(x-\beta)^5}{30}\right]_\alpha^\beta = \frac{a(\beta-\alpha)^5}{30}$$

$$= \frac{a(\beta-\alpha)^5}{\lambda} = \frac{\lambda}{5} = 6.$$

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BRUSH UP *for* JEE Main & Advanced 2022

Brush up your concepts to get high rank in JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year according to the requirements of these Entrance Tests.

Sequences and Series

SEQUENCE

Sequence is a function whose domain is sub-set of N, the set of natural numbers. Sequence gives the images of 1, 2, 3, ..., n as $f_1, f_2, f_3, \dots, f_n$ where $f_n = f(n)$. The function $f(n) = (-1)^{n-1}$ gives the sequence whose terms are alternatively 1 and -1. The function $f(n) = n^3$ gives the sequence $1^3, 2^3, 3^3, \dots, n^3$ whose domain is 1, 2, 3, ..., n and images $f(1) = 1$, $f(2) = 2^3$, ..., so on.

Thus a mapping of $f : N \to C$ defined by $f(n) = t_n$, $n \in N$ is called a sequence to be denoted by

$$\{f(1), f(2), \dots\} = \{t_1, t_2, t_3, \dots\} = \{t_n\}$$

Thus, 1, 3, 5, 7, ...; 2, 4, 6, 8, ... ; 1, 4, 16, 64, ... ; 27, 9, 3, 1, 1/3, ... are sequences.

Note : Sequence is said to be finite or infinite sequence according as it has finite or infinite number of terms or according as the domain of function is finite subset or an infinite subset of N.

SERIES

By adding or subtracting the terms of a sequence gives a series. Same as sequence, series is said to be finite or infinite series according it has finite or infinite number of terms.

1 + 3 + 5 + 7 + ..., 2 + 4 + 6 + 8 + ...,

$1 - 4 + 16 - 64 + \dots$ and $27-9+3-1+\frac{1}{3}\dots$, are series.

The sequence $1^3, 2^3, 3^3, \dots, n^3$ gives the series

$1^3 + 2^3 + 3^3 + \dots + n^3$ or $1+\frac{1}{2^3}+\frac{1}{3^3}+\dots$

PROGRESSION

Progression is a sequence whose terms are arranged by a definite rule.

There are three types of progressions (1) Arithmetic progression (A.P.), (2) Geometric progression (G.P.) and (3) Harmonic progression (H.P.).

ARITHMETIC PROGRESSION (A.P.)

A sequence of numbers $< t_n >$ is said to be an arithmetic progression (A.P.) if difference between any two consecutive terms is same. If t_{n-1} and t_n are two consecutive terms of A.P. then $t_n - t_{n-1}$ is a constant $\forall\, n \in N$. This constant is called the common difference of A.P. and is usually denoted by d.

If a is the first term and d is the common difference, then an A.P. can be represented by a, $a + d$, $a + 2d$, $a + 3d$, ...

General Term of A.P.

The general term of an A.P. is denoted by t_n, called n^{th} term and given by $t_n = a + (n - 1)d$.

- The r^{th} term from the end is $(n - r + 1)^{\text{th}}$ term from the beginning.
  $\therefore\ t_{n-r+1} = a + (n - r)d$
- If last term of an A.P. is l, then r^{th} term from the end is $l - (r - 1)d$.

Sum of First *n* Terms of an A.P.

The sum of n terms of the A.P. $a + (a + d) + (a + 2d) + \dots + a + (n - 1)d$ is given by

$$S_n=\frac{n}{2}[2a+(n-1)d]=\frac{n}{2}[a+\overline{a+(n-1)d}]=\frac{n}{2}(a+l),$$

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where l is the last term of the A.P. and given by $l = a + (n - 1)d = t_n$

Arithmetic Mean (A.M.)

- If a and b are two real numbers then their arithmetic mean *i.e.*, A.M. is given by $\dfrac{a+b}{2}$.
- If a, x, b are in A.P., then $x = \dfrac{a+b}{2}$ is the A.M. of a and b.
- If $a, x_1, x_2, x_3, ..., x_n, b$ are in A.P., then $x_1, x_2, ..., x_n$ are n A.M.'s between a and b, then $d = \dfrac{b-a}{n+1}$.

  $$\therefore \quad x_1 = a + d = \frac{na+b}{n+1},\ x_2 = a + 2d = \frac{(n-1)a+2b}{n+1},$$

  $$x_3 = a + 3d = \frac{(n-2)a+3b}{n+1},\ .....$$

  $$....., x_n = b - d = \frac{a+nb}{n+1} \text{ or } a + \frac{n(b-a)}{n+1} = a + nd$$

Properties of A.P.

- Three numbers a, b, c are in A.P. if and only if $2b = a + c$.
- If the number of terms of an A.P. is even, then A.M. of two middle most terms is A.M. of first and last terms.
  i.e., if $a_1, a_2, a_3, ..., a_{n-1}, a_n, a_{n+1}, a_{n+2}, ..., a_{2n-1}, a_{2n}$ are in A.P. then $a_1 + a_{2n} = a_n + a_{n+1}$
  Moreover, $a_1 + a_{2n} = a_2 + a_{2n-1} = ... = a_n + a_{n+1}$
  i.e., sum of the first and last term is equal to the sum of the terms equidistant from beginning and end.
- If the number of terms of an A.P. is odd, then sum of its first and last term is twice the middle term.
  i.e., if $a_1, a_2, ..., a_{n-1}, a_n, a_{n+1}, a_{n+2}, ..., a_{2n+1}$ are in A.P., then $a_1 + a_{2n+1} = 2 \cdot a_{n+1}$
  moreover, $a_1 + a_{2n+1} = a_2 + a_{2n} = ...$
  $= a_n + a_{n+2} = 2a_{n+1}$
  Also, $a_1 + a_2 + a_3 + ... + a_{2n+1}$
  $=$ Number of terms $\times\, a_{n+1}$
  i.e., $S_{2n+1} = (2n + 1)a_{n+1}$
  e.g., 1, 3, 5, 7, 9, 11, 13 are in A.P., then
  $S_7 = 7 \times a_4 = 7 \times 7 = 49$.
- If the terms of an A.P. are chosen at regular intervals, then the new sequence so formed also forms an A.P.
- If $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ are two A.P.'s, then $a_1 \pm b_1, a_2 \pm b_2, ..., a_n \pm b_n$ are also in A.P. with common difference $d_1 \pm d_2$, where d_1 and d_2 respectively are the common difference of the given A.P.'s.
- If $S_1, S_2, S_3, ..., S_n$ denote the sum of first term, sum of first two terms, sum of first three terms, ..., the sum of n terms respectively of an A.P., then
  (a) $t_1 = S_1$
  (b) common difference, $d = S_2 - 2S_1$
  (c) $t_n = S_n - S_{n-1}$

Invariance of A.P.

If a, b, c are in A.P. with common difference d, then c, b, a ; $a + \alpha, b + \alpha, c + \alpha$; $a - \alpha, b - \alpha, c - \alpha$; $\alpha a, \alpha b, \alpha c$ and $\dfrac{a}{\alpha}, \dfrac{b}{\alpha}, \dfrac{c}{\alpha}$ are also in A.P. with common difference $-d, d, d, \alpha d, \dfrac{d}{\alpha}$ respectively, where α is a fixed non-zero real number.

- For odd number of terms in A.P., select (take) the terms as
  .... $a - 2d, a - d, a, a + d, a + 2d$, ...
  (common difference d)
- For even number of terms in A.P., take the terms as
  ..., $a - 3d, a - d, a + d, a + 3d$, ...
  (common difference $2d$)

Special Series or Identities in Natural Numbers

- $1+2+3+....+n = \displaystyle\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
- $1^2+2^2+3^2+...+n^2 = \displaystyle\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3+2^3+3^3+...+n^3 = \displaystyle\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $1+3+5+....+(2n-1) = \displaystyle\sum_{r=1}^{n}(2r-1) = n^2, t_r = 2r-1$
- $1^2+3^2+5^2+....$ to n terms $= \dfrac{n(4n^2-1)}{3}$

  $$\left[\because t_n = (2n-1)^2 = 4n^2 - 4n + 1 \Rightarrow \Sigma t_n = \frac{n(4n^2-1)}{3}\right]$$

General Term for Some Special Series or Sequences and their Sums

$1n, 2(n - 1), 3(n - 2), ..., (n - 1)2, n(1)$

$\therefore \quad t_r = r(n - \overline{r-1})$

$$\therefore \quad S_n = \sum_{r=1}^{n} r(n-r+1) = (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$$

$$= \frac{(n+1)n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(n+2)}{6}$$

GEOMETRIC PROGRESSION (G.P.)

A sequence in which the ratio of any two terms is same *i.e.,* if the ratio of its each term to its preceding term is always a fixed quantity (number) is called G.P. The fixed quantity is called common ratio denoted by r. If a be the first term, then $a, ar, ar^2, \ldots., ar^{n-1}$ is a sequence of G.P.

General Term of G.P.

- The general term of any sequence is usually denoted by t_n, which is the n^{th} term.

  $\therefore$ n^{th} term of G.P., $a, ar, ar^2, \ldots.$ is

  $$t_n = ar^{n-1}, \text{ where } r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \ldots. = \frac{t_n}{t_{n-1}}$$

- If a G.P. having n terms, then R^{th} term from the end $= (n - R + 1)^{\text{th}}$ term from the starting (beginning) $= ar^{n-R}$ and the R^{th} term from the end of a G.P. whose last term is l and common ratio r is $l \cdot \left(\frac{1}{r}\right)^{R-1}$.

Sum of First *n* Terms of a G.P.

If a be the first term and r be the common ratio, then sum of the terms $a, ar, ar^2, \ldots, ar^{n-1}$ is given by

$S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a-lr}{1-r}$, where l is the last term and $|r| < 1$ *i.e.,* $-1 < r < 1$

and $S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{lr-a}{r-1}$, where l is the last term and $|r| > 1$, *i.e.,* $r < -1$ or $r > 1$

If $r = 1$, then $a, ar, ar^2, \ldots., ar^{n-1}$ reduces to $a, a, a, \ldots., a$ (n factors) then $S_n = n \cdot a$.

Sum of Infinite Terms of G.P.

- $S_\infty = \dfrac{a}{1-r}$ when $|r| < 1$ *i.e.,* $-1 < r < 1$
- S_∞ does not exist if r is either 1 or greater than 1.

Geometric Mean (G.M.)

- If a, x, b are in G.P. then x is called G.M. between a and b and given by $x = \sqrt{ab}, a > 0, b > 0$.
- If $a, x_1, x_2, \ldots., x_n, b$ are in G.P. then $x_1, x_2, x_3, \ldots., x_n$ are the n geometric means between a and b and if r is common ratio, then terms $x_1, x_2, x_3, \ldots, x_n$ reduces to $ar, ar^2, \ldots, ar^n$ where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ then

  $$x_1 = ar = \text{first G.M.} = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

  $$x_2 = ar^2 = \text{second G.M.} = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

  ………………………………

  $$x_n = ar^n \text{ or } \frac{b}{r} = b\left(\frac{a}{b}\right)^{\frac{1}{n+1}} \text{ or } a \cdot \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Properties of G.P.

- If we take reciprocal of the terms $a, ar, ar^2, \ldots, ar^{n-1}, \ldots$ we get the new sequence $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \ldots., \frac{1}{ar^{n-1}}, \ldots$ then the sequence remains in G.P. with common ratio $1/r$.
- In a finite G.P. the product of term equidistant from the starting (beginning) and the end is always the same and is equal to the product of first and last term *i.e.,* if terms of G.P. are $a_1, a_2, \ldots., a_n$ then $a_1 a_n = a_2 a_{n-1}, = a_3 a_{n-2} = \ldots. = a_R\, a_{n-R+1}$.
- If $a_1, a_2, a_3, \ldots., a_n$ are in G.P. ($a_i > 0$, $i = 1, 2, 3, \ldots, n$ and $a_i \neq 0$), then $\log a_i$ *i.e.,* $\log a_1, \log a_2, \log a_3, \ldots., \log a_n \ldots$ are in A.P. and vice versa.
- If the first term of a G.P. of n terms is a with last term l, then product of all the terms of G.P. is $(al)^{n/2}$

  i.e., $a, ar, ar^2, \ldots., ar^{n-1} = a^n r^{\frac{n(n-1)}{2}} = (a^2)^{\frac{n}{2}} (r^{n-1})^{\frac{n}{2}}$

  $= (a^2 r^{n-1})^{n/2} = (a \cdot ar^{n-1})^{n/2} = (al)^{n/2}$
- If a_i and b_i $(i = 1, 2, \ldots, n)$ are two G.P.'s with common ratio r_1 and r_2 respectively, then

  (a) $a_1b_1, a_2b_2, a_3b_3, \ldots., a_nb_n$ is a G.P. whose common ratio is r_1r_2 $(\neq 0)$

  (b) $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots., \frac{a_n}{b_n}$ is a G.P. whose common ratio is $\frac{r_1}{r_2}$ $(r_1, r_2 \neq 0)$
- If a, b, c are in G.P. then square of middle term = product of extreme terms. *i.e.,* $b^2 = ac$
- If p^{th}, q^{th} and r^{th} terms of a G.P. are in G.P. then p, q, r are in A.P.
- If a, b, c are in A.P. and G.P. both, then each term is identical *i.e.,* $a = b = c$.
- If a, b, c are in A.P. then $\alpha^a, \alpha^b, \alpha^c$ $(\alpha \neq \pm 1)$ will be in G.P.
- Let the first term of a G.P. is positive if $r > 1$, then it is increasing G.P. but if $0 < r < 1$ then it is decreasing G.P.
- Let the first term of a G.P. is negative, if $r > 1$, then it is a decreasing G.P. but for $0 < r < 1$ it is increasing G.P.

Invariance of G.P.

- If a, b, c are in G.P. with common ratio r then c, b, a ; $\alpha a, \alpha b, \alpha c$; $\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}$ $(\alpha \neq 0)$ are in G.P. with common ratio $\frac{1}{r}, r, r$ respectively.
- If a, b, c are in G.P. with common ratio r then a^m, b^m, c^m, $m \in N$ also in G.P. with common ratio r^m.

HARMONIC PROGRESSION (H.P.)

A sequence is an H.P. if the reciprocal of its terms form an A.P. *e.g.*, $\frac{3}{2}, 1, \frac{3}{4}$ are in H.P. since $\frac{2}{3}, 1, \frac{4}{3}$ are in A.P.

General Term of H.P.

As we know that $a, a + d, a + 2d, ..., a + (n - 1)d, ...$ are terms of A.P.

$\therefore$ Terms of H.P. be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, ..., \frac{1}{a+(n-1)d}$

$\therefore$ General term t_n of H.P. is $\frac{1}{a+(n-1)d}$.

Harmonic Mean (H.M.)

- If a, x, b are in H.P., then $x = \frac{2ab}{a+b}$ is called the harmonic mean between a and b.
- The harmonic mean of n non zero numbers $a_1, a_2, a_3, ..., a_n$ is given by $\frac{1}{H} = \frac{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+...+\frac{1}{a_n}}{n}$
- If a and b be two given numbers and set of n numbers *i.e.*, harmonic means $H_1, H_2, H_3, ..., H_n$ be inserted between these two numbers then $a, H_1, H_2, H_3, ..., H_n, b$ are in H.P.

  $\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, ..., \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

  This A.P. has $(n + 2)$ terms. Let D be the common difference, then $(n + 2)^{\text{th}}$ term is given by

  $$t_{n+2} = \frac{1}{b} = \frac{1}{a} + (n+1)D \Rightarrow D = \frac{a-b}{(n+1)ab}$$

  $$\therefore H_1 = \frac{(n+1)ab}{a+nb}, H_2 = \frac{(n+1)ab}{2a+(n-1)b}, ...$$

  $$H_n = \frac{(n+1)ab}{na+b}$$

  or $\frac{1}{H_n} = \frac{1}{a} + nD = \frac{1}{a} + \frac{n(a-b)}{(n+1)ab} = \frac{na+b}{(n+1)ab}$

Properties of H.P.

- No term of H.P. can be zero and there is no general formula for calculating the sum of n terms of H.P.
- If H is the H.M. between two numbers a and b then
  (a) $(H - 2a)(H - 2b) = H^2$
  (b) $\frac{H+a}{H-a} + \frac{H+b}{H-b} = 2$
- If $x_1, x_2, x_3, ..., x_n$ are in H.P., then $x_1x_2 + x_2x_3 + x_3x_4 + ... + x_{n-1}x_n = (n - 1)\, x_1\, x_n$.

Invariance of H.P.

If a, b, c are in H.P., then c, b, a; $\alpha a, \alpha b, \alpha c$; $\frac{a}{\alpha}, \frac{b}{\alpha}, \frac{c}{\alpha}$ $(\alpha \neq 0)$ are also in harmonic progression.

Formation of Progression

- Two consecutive terms determine the required progression.

  $a, b \Rightarrow a, b, 2b - a \in$ A.P.

  $a, b \Rightarrow a, b, \frac{b^2}{a} \in$ G.P.

  $a, b \Rightarrow a, b, \frac{ab}{2a-b} \in$ H.P.
- If a, b, c are in A.P. and b, c, a are in G.P. then c, a, b are in H.P.

Relationship between A.M., G.M. and H.M.

a, A.M., $b \in$ A.P., a, G.M., $b \in$ G.P. and a, H.M., $b \in$ H.P.
Let A.M. $= A$, G.M. $= G$ and H.M. $= H$

Now, $a, A, b \in \text{A.P.} \Rightarrow A = \frac{a+b}{2}$

Similarly, $G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$

$$\therefore A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0$$

$\Rightarrow A \geq G$ ... (i)

$$\text{And } G - H = \sqrt{ab} - \frac{2ab}{a+b} = \sqrt{ab}\left(\frac{a+b-2\sqrt{ab}}{a+b}\right)$$

$$= \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b} \geq 0$$

$\Rightarrow G \geq H$ ... (ii)

Thus, from (i) and (ii), we have $A \geq G \geq H$, equality holds only when $a = b$.

- A, G, H form a G.P. $\Rightarrow G^2 = AH$.
- The equation having a, b as its roots is $x^2 - (a + b)x + ab = 0$

$$\Rightarrow x^2 - 2Ax + G^2 = 0 \left(\text{using } A = \frac{a+b}{2}, G = \sqrt{ab}\right)$$

⇒ Roots a, b are $\dfrac{2A\pm\sqrt{(4A^2-4G^2)}}{2}$

i.e., $A+\sqrt{A^2-G^2},\ A-\sqrt{A^2-G^2}$

- If A, G, H are A.M., G.M. and H.M. between the three numbers a, b, c respectively then equation whose roots are a, b, c is given by
  $x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0$
  $$\Rightarrow\ x^3-3Ax^2+\frac{3G^3}{H}x-G^3=0$$
  $$\left\{\begin{array}{l}\text{where } A=\dfrac{a+b+c}{3},\ G=(abc)^{1/3}\\ \text{and } \dfrac{1}{H}=\dfrac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}\end{array}\right\}$$
- If A, G, H be A.M., G.M., H.M. between the numbers a and b then
  $$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\left\{\begin{array}{l}A=\dfrac{a+b}{2},\text{ when } n=0\\ G=\sqrt{ab},\text{ when } n=-\dfrac{1}{2}\\ H=\dfrac{2ab}{a+b}\ \text{ when } n=-1\end{array}\right\}$$
- If two A.M.'s A_1 and A_2, two G.M.'s G_1 and G_2 and two H.M.'s H_1 and H_2 are inserted between any two numbers (say a and b) then
  $$\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$$
- If the number of terms of any A.P./G.P./H.P. is odd, then A.M./G.M./H.M of first and last term is middle term of the series respectively.
- If the number of terms of any A.P./G.P./H.P. is even, then A.M./G.M./H.M. of middle two terms is A.M./G.M./H.M. of the first and last terms respectively.

ARITHMETICO-GEOMETRIC PROGRESSION (A.G.P.)

The collection of A.P. and G.P. is said to be A.G.P.

If $a_1, a_2, a_3, ..., a_n$ is an A.P. with common difference d and $b_1, b_2, b_3, ..., b_n$ is a G.P. with common ratio r, then the sequence $a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n$ is said to be an arithmetico-geometric sequence.

Now $a_1, a_2, a_3, ..., a_n$ becomes $a_1 = a$(say), $(a + d)$, $(a + 2d)$, ... $a + (n - 1)d$ and $b_1, b_2, b_3, ..., b_n$ becomes $b_1 = b$(say), $br, br^2, ..., br^{n-1}$ then $a_1b_1, a_2b_2, a_3b_3, ..., a_nb_n$ becomes $ab, (a + d)br, (a + 2d)br^2, ..., (a + (n - 1)d)br^{n-1}$ and $a, (a + d)r, (a + 2d)r^2, ..., [a + (n - 1)d]r^{n-1}$ for $b = 1$.

General Term of A.G.P.

As the terms of A.G.P. are a, $(a + d)r$, $(a + 2d)r^2$, ...

∴ $t_n = [a + (n - 1)d]r^{n-1}$.

Sum of A.G.P.

The sum of n terms of an A.G.P. with terms a, $(a + d)r$, $(a + 2d)r^2$ is given by

- $$S_n=\left\{\frac{a}{1-r}+\frac{dr(1-r^{n-1})}{(1-r)^2}-\frac{\{a+(n-1)d\}r^n}{1-r},\ r\neq 1\right\}$$
- Sum to infinite terms of a A.G.P. when $|r| < 1$ *i.e.*, $-1 < r < 1$ then $r^n \to 0$.
  $$S_\infty=\frac{a}{1-r}+\frac{dr}{(1-r)^2}=\frac{a+(d-a)r}{(1-r)^2}$$

Properties of A.G.P.

A.G.P. is given by a, $(a + d)r$, $(a + 2d)r^2$, $(a + 3d)r^3$, ...

- If $r = 1$, then A.G.P. reduces to A.P. with terms a, $a + d$, $a + 2d$, ...
- If $d = 0$, then A.G.P. reduces to G.P. with terms a, ar, ar^2, ...

Method to Find the General Term (t_n) of G.P. and A.G.P. by Using Successive Order of Difference

If few terms of a series are given and if p^{th} order of differences of these terms form a G.P., with common ratio r, then general form of the two series is taken as

$$t_n = ar^{n-1} + f(n),$$

where $f(n)$ is a rational integral function of n of $(p - 1)$ dimensions.

∴ Consider the general term as follows (given):

$t_n = ar^{n-1}$ for direct G.P.

$t_n = ar^{n-1} + b$ if first order successive difference is in G.P.

$t_n = ar^{n-1} + bn + c$ if second order successive difference is in G.P.

$t_n = ar^{n-1} + bn^2 + cn + d$

if third order successive difference is in G.P., where a, b, c, d, ... are constants which can be obtained by using t_1, t_2, t_3, ... by putting $n = 1, 2, 3$, ... respectively in above relations.

V_n Method

The method is used to find the sum of the series of the forms

(A) $a_1a_2 \dots a_r + a_2a_3 \dots a_{r+1} + \dots + a_na_{n+1} \dots a_{n+r-1}$

(B) $\dfrac{1}{a_1a_2\dots a_r}+\dfrac{1}{a_2a_3\dots a_{r+1}}+\dots+\dfrac{1}{a_n\,a_{n+1}\dots a_{n+r-1}}$,

where $a_1, a_2, ..., a_n \in$ A.P.

In order to find the sum of terms of the series (A) we use the following steps.

Step-I : Let the common difference of A.P. = d.

$\therefore\quad a_n = a_1 + (n - 1)d$

Let S_n denotes the sum of n terms and t_n = n^{th} term of the series. Define, $t_n = a_n\, a_{n+1} \ldots a_{n+r-1}$.

Step-II : Define V_n from t_n by taking next term of the series (*i.e.* one extra term in t_n for V_n)

i.e., $V_n = t_n \cdot a_{n+r} = a_n\, a_{n+1} \ldots .\, a_{n+r-1}\, a_{n+r}$

$\therefore\quad V_{n-1} = a_{n-1}\, a_n \ldots .\, a_{n+r-2}\, a_{n+r-1}$

(replacing n by $(n - 1)$ in V_n)

Step-III : Make the difference between V_n and V_{n-1} to develop the relationship between t_n, V_n and V_{n-1}.

$$\therefore\quad V_n - V_{n-1} = (a_n\, a_{n+1}\, a_{n+2} \ldots a_{n+r-1})(a_{n+r} - a_{n-1})$$

$$\Rightarrow\quad V_n - V_{n-1} = t_n[a_1 + (n + r - 1)d - a_1 - (n - 2)d]$$

$$\Rightarrow\quad V_n - V_{n-1} = (r + 1)d \cdot t_n$$

$$\Rightarrow\quad t_n = \frac{1}{(r+1)d}[V_n - V_{n-1}]$$

Now put n = 1, 2, ..., n to get t_1, t_2, ..., t_n and add them columnwise to obtain

$$S_n = t_1 + t_2 + \ldots + t_n = \frac{1}{(r+1)d}[V_n - V_0]$$

$$\Rightarrow\; S_n = \frac{1}{(r+1)(a_2 - a_1)}(a_n\, a_{n+1} \ldots a_{n+r} - a_0 a_1 a_2 \ldots a_r)$$

where $a_0 = a_1 - d$, which is required sum of n terms of the series of form (A).

Also for series (B), we have

$$t_n = \frac{1}{a_n\, a_{n+1} \ldots .\, a_{n+r-2} a_{n+r-1}}$$

$$\therefore\; V_n = \frac{1}{a_{n+1}\, a_{n+2} \ldots .\, a_{n+r-2}\, a_{n+r-1}}$$

(Avoiding first term from t_n)

$$\text{and } V_{n-1} = \frac{1}{a_n a_{n+1} \ldots .\, a_{n+r-3} a_{n+r-2}}$$

(Replacing n by $(n - 1)$ in V_n)

$$\therefore\quad V_n - V_{n-1} = \frac{a_n - a_{n+r-1}}{a_n\, a_{n+1} \ldots a_{n+r-1}} = t_n(1-r)\,d$$

$$\Rightarrow\quad t_n = \frac{1}{d(r-1)}[V_{n-1} - V_n]$$

Now, put n = 1, 2, ..., n and add columnwise to get

$$S_n = t_1 + t_2 + \ldots . + t_n = \frac{1}{(r-1)d}(V_0 - V_n),$$

where V_0 is obtained by putting n = 1 in V_{n-1}.

Method to Find $\sum_{r=1}^{n}\frac{1}{t_r}$ when $\sum_{r=1}^{n} t_r$ is Known

We discuss the method with the help of an example.

$$\text{Let } \sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)}{3} = S_n$$

$$\therefore\; t_n = S_n - S_{n-1} = n(n+1)$$

$$\therefore\; \frac{1}{t_n} = \frac{1}{n(n+1)}$$

$$\text{Let } V_n = \frac{1}{n+1} \quad \text{(Avoiding first term of } t_n) \qquad \ldots\text{(i)}$$

$$\therefore\; V_{n-1} = \frac{1}{n} \quad \text{(Replacing } n \text{ by } (n-1) \text{ in } V_n) \qquad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$V_n - V_{n-1} = -\frac{1}{n(n+1)} = -\frac{1}{t_n} \Rightarrow \frac{1}{t_n} = V_{n-1} - V_n \quad \ldots(*)$$

Now putting n = 1, 2, 3, ..., n in (*) and adding columnwise we get

$$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \ldots + \frac{1}{t_n} = (V_0 - V_n) = 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

$$\therefore\; \sum_{r=1}^{n} \frac{1}{t_r} = \frac{n}{n+1}$$

$$\text{and } \lim_{n\to\infty} \sum \frac{1}{t_r} = \lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$$

To Find the Coefficients of Various Exponents of the Variable in the Product of Linear Factors

$(x + 1)(x + 2)(x + 3) \ldots (x + n)$ (n linear factors)

$= x^n$ + (sum of constant numbers)x^{n-1}

+ (sum of the product of numbers taken 2 at a time)x^{n-2}

+ (sum of the product of numbers taken 3 at a time)x^{n-3}

+ .... + (sum of the product of numbers taken $(n - 1)$ at a time)x^1 + product of the numbers taken all at a time

$$= x^n + (1+2+3+\ldots+n)x^{n-1}$$

$$+\frac{1}{2}[(1+2+3+\ldots+n)^2 - (1^2 + 2^2 + \ldots + n^2)]\, x^{n-2}$$

$$+\frac{(n-2)(n-1)\, n^2 (n+1)^2}{48} x^{n-3} + \ldots .$$

$$+ [2 \cdot 3 \ldots (n - 1)n + \ldots + 1 \cdot 3 \cdot 4 \ldots . (n - 1)n + \ldots . + 1 \cdot 2 \cdot 3 \ldots (n - 1)]\, x^1 + 1 \cdot 2 \cdot 3 \ldots n x^0$$

$\therefore\quad$ Coefficient of x^n = 1

CONCEPT MAP

CIRCLES

Class XI

CIRCLE

A circle is defined as the locus of a point, which moves in a plane such that its distance from a fixed point in that plane is always constant. The fixed point is called the centre and the constant distance is called the radius of the circle.

General Equation of Circle

General equation of circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $(C) = (-g, -f)$ and radius $(r) = \sqrt{g^2 + f^2 - c}$.

Standard Equation of Circle

Standard equation of circle is given by $(x - h)^2 + (y - k)^2 = a^2$ with centre $(C) = (h, k)$ and radius $(r) = a$.

Second Degree Equation of Circle

The second degree equation given by $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, represents a circle if $a = b$, $h = 0, \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ and $g^2 + f^2 - ac > 0$.

Parametric Form of Circle

$x = -g + r\cos\theta$, $y = -f + r\sin\theta$, where $r = \sqrt{g^2 + f^2 - c}$ represents the parametric equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

Simplest Form of Circle

Simplest form of circle is $x^2 + y^2 = a^2$, where centre $(C) = (0, 0)$ and radius, $(r) = a$

Diametric Form of Circle

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, represents equation of circle with diameter AB, where $A(x_1, y_1)$ and $B(x_2, y_2)$ are end points of diameter.

Equation of Director Circle

Equation of director circle of the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.

Equation of Radical Axis of Two Circle

Equation of radical axis of two circles $S_1 = x^2 + y^2 + 2gx + 2fy + c = 0$ and $S_2 = x^2 + y^2 + 2g'x + 2f'y + c' = 0$ is given by $S_1 - S_2 = 0$.

Angle of Intersection between Two Circles

The angle between two intersecting circles having centres at C_1 & C_2 and radii r_1 & r_2 respectively is given by $\cos\theta = \dfrac{r_1^2 + r_2^2 - (C_1C_2)^2}{2r_1r_2}$.

Orthogonal Circles

Two circles are orthogonal iff $2gg' + 2ff' = c + c'$.

Equation of Tangent & Normal to the Circle

Equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at point (x_1, y_1) is given by $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

Equation of normal at point $(x_1, y_1,)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $\dfrac{(x - x_1)}{x_1 + g} = \dfrac{(y - y_1)}{y_1 + f}$.

CONCEPT MAP

INVERSE TRIGONOMETRIC FUNCTIONS

Class XII

INVERSE TRIGONOMETRIC FUNCTIONS

Inverse trigonometric functions are the inverse function of the trigonometric functions, specifically, they are the inverse of the sine, cosine, tangent, cotangent, secant and cosecant functions. They are also termed as anti-trigonometric functions or cyclometric functions.

Function	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\cot^{-1}x$	$\text{cosec}^{-1}x$	$\sec^{-1}x$
Domain	$[-1, 1]$	$[-1, 1]$	R	R	$R - (-1, 1)$	$R - (-1, 1)$
Range	$[-\pi/2, \pi/2]$	$[0, \pi]$	$(-\pi/2, \pi/2)$	$(0, \pi)$	$[-\pi/2, \pi/2] - \{0\}$	$[0, \pi] - \{\pi/2\}$

- $\sin^{-1}(-x) = -\sin^{-1}x, x \in [-1, 1]$
- $\cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1, 1]$
- $\tan^{-1}(-x) = -\tan^{-1}x, x \in R$
- $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in R$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x, |x| \geq 1$
- $\sec^{-1}(-x) = \pi - \sec^{-1}x, |x| \geq 1$
- $\sin^{-1}x + \cos^{-1}x = \pi/2, x \in [-1, 1]$
- $\tan^{-1}x + \cot^{-1}x = \pi/2, x \in R$
- $\text{cosec}^{-1}x + \sec^{-1}x = \pi/2, |x| \geq 1$

- $3\sin^{-1}x = \begin{cases} \sin^{-1}(3x - 4x^3), \text{ if } -\frac{1}{2} \leq x \leq \frac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3), \text{ if } \frac{1}{2} < x \leq 1 \\ -\pi - \sin^{-1}(3x - 4x^3), \text{ if } -1 \leq x < -\frac{1}{2} \end{cases}$
- $3\cos^{-1}x = \begin{cases} \cos^{-1}(4x^3 - 3x), \text{ if } \frac{1}{2} \leq x \leq 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x), \text{ if } -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x), \text{ if } -1 \leq x \leq -\frac{1}{2} \end{cases}$
- $3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } x < -\frac{1}{\sqrt{3}} \end{cases}$

- $\sin^{-1}(1/x) = \text{cosec}^{-1}x; x \geq 1 \text{ or } x \leq -1$
- $\cos^{-1}(1/x) = \sec^{-1}x; x \geq 1 \text{ or } x \leq -1$
- $\tan^{-1}(1/x) = \cot^{-1}x; x > 0$
- $\sin(\sin^{-1}x) = x \;\forall\; x \in [-1, 1]$
- $\cos(\cos^{-1}x) = x \;\forall\; x \in [-1, 1]$
- $\tan(\tan^{-1}x) = x \;\forall\; x \in R$
- $\cot(\cot^{-1}x) = x \;\forall\; x \in R$
- $\text{cosec}(\text{cosec}^{-1}x) = x \;\forall\; x \in R - (-1, 1)$
- $\sec(\sec^{-1}x) = x \;\forall\; x \in R - (-1, 1)$

- $2\sin^{-1}x = \begin{cases} \sin^{-1}(2x\sqrt{1 - x^2}), |x| \leq \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1 - x^2}), \frac{1}{\sqrt{2}} \leq x \leq 1 \\ -\pi - \sin^{-1}(2x\sqrt{1 - x^2}), -1 \leq x \leq \frac{-1}{\sqrt{2}} \end{cases}$
- $2\cos^{-1}x = \begin{cases} \cos^{-1}(2x^2 - 1), 0 \leq x \leq 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), -1 \leq x \leq 0 \end{cases}$
- $2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1 - x^2}\right), |x| < 1 \\ \sin^{-1}\left(\frac{2x}{1 + x^2}\right), |x| \leq 1 \\ \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 \leq x \leq 1 \end{cases}$

- $\sin^{-1}x \pm \sin^{-1}y$

  $= \begin{cases} \sin^{-1}(x\sqrt{1 - y^2} \pm y\sqrt{1 - x^2}) \;\forall\; x, y \geq 0, x^2 + y^2 \leq 1 \\ \pi - \sin^{-1}(x\sqrt{1 - y^2} \pm y\sqrt{1 - x^2}) \;\forall\; x, y \geq 0, x^2 + y^2 > 1 \end{cases}$
- $\cos^{-1}x \pm \cos^{-1}y$

  $= \begin{cases} \cos^{-1}(xy \mp \sqrt{1 - x^2}\sqrt{1 - y^2}) \;\forall\; x, y \geq 0, x^2 + y^2 \leq 1 \\ \pi - \cos^{-1}(xy \mp \sqrt{1 - x^2}\sqrt{1 - y^2}) \;\forall\; x, y \geq 0, x^2 + y^2 > 1 \end{cases}$
- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right), xy < 1$
- $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right), xy > -1$

Coefficient of $x^{n-1} = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

Coefficient of x^{n-2}

$$=\frac{1}{2}\left[\frac{n^2(n+1)^2}{4}-\frac{n(n+1)(2n+1)}{6}\right]=\frac{n(n^2-1)(3n+2)}{24}$$

Constant term = $1 \cdot 2 \cdot 3 \cdot 4 .... n = n!$

PROBLEMS

Single Correct Answer Type

1. If $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+....\infty=\frac{\pi^2}{6}$ then the value of $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+...\infty$ equals

(a) $\frac{\pi^2}{12}$ (b) $\frac{\pi^2}{24}$ (c) $\frac{\pi^2}{48}$ (d) $\frac{\pi^2}{32}$

2. If $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+....+\frac{1}{10\cdot11\cdot12}=\frac{m}{n}$ (reduced fraction), then sum of the digits of $(m + n)$ equals

(a) 12 (b) 14 (c) 329 (d) 264

3. The natural numbers are grouped as follows {1}, {2, 3, 4}, {5, 6, 7, 8, 9}, {10, 11, 12, 13, 14, 15, 16}, ... then the mean of first term and last term of the n^{th} group is

(a) $n^2 + n + 1$ (b) $n^2 + n - 1$

(c) $n^2 - n + 1$ (d) None of these

4. The number $2^{2\sin 2x-1}$, 3, $2^{4-2\sin 2x}$ form first three terms of an A.P., then its 2018th term equals

(a) 2020 (b) 2019 (c) 2018 (d) 2017

5. The coefficient of x^{16} in the product $(1 - x)(1 - 3x)(1 - 3^2x)(1 - 3^3x)$ ......$(1 - 3^{16}x)$ is

(a) $3^{120}\left(\frac{3^{17}-1}{2}\right)$ (b) $3^{120}\left(\frac{3^{16}-1}{2}\right)$

(c) $3^{120}(3^{16} - 1)$ (d) $3^{120}(3^{17} - 1)$

6. Let $f_n(S) = SSS.....S$ (n digits), where $S = 1, 2, 3, ..., 9$ then $(f_n(6))^2 + f_n(8)$ equals to

(a) $f_{2n}(1)$ (b) $4f_{2n}(1)$

(c) $(f_{2n}(2))^2$ (d) $-4f_{2n}(1)$

7. If 4 arithmetic means, 4 geometric means and 4 harmonic means are inserted between 2 and 12, then the cubic equation whose roots are first A.M., second G.M. and third H.M. between 2 and 12 is

(a) $x^3 - (8 + 2\cdot6^{2/5})x^2 + 16(1 + 6^{2/5})x - 32\cdot6^{2/5} = 0$

(b) $x^3 + (8 + 2\cdot6^{2/5})x^2 + 16(1 + 6^{2/5})x + 32\cdot6^{2/5} = 0$

(c) $x^3 + (8 + 2\cdot6^{2/5})x^2 - 16(1 + 6^{2/5})x - 32\cdot6^{2/5} = 0$

(d) $x^3 - (8 - 2\cdot6^{2/5})x^2 + 16(1 - 6^{2/5})x + 32\cdot6^{2/5} = 0$

8. Let t_n be the n^{th} term of a G.P. of positive numbers and $\alpha=\sum_{n=1}^{2019} t_{2n}$ and $\beta=\sum_{n=1}^{2019} t_{2n-1}$ such that $\alpha \neq \beta$, then common ratio of the G.P. is

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\frac{\beta}{\alpha}}$ (d) $\sqrt{\frac{\alpha}{\beta}}$

9. The sum of the infinite series

$\frac{1\cdot2}{3}+\frac{2\cdot3}{3^2}+\frac{3\cdot4}{3^3}+\frac{4\cdot5}{3^4}+.....\infty$ is

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{9}{4}$ (d) $\frac{8}{9}$

10. The sum of infinite terms of a decreasing geometric series with common ratio 'r' is $16k$ such that $|r| < 1$, $r \neq 0$. The ratio of the 10th term to the 8th term is 1/25 and the ratio of the third term to the square of the second term is 1/64, then k equals to

(a) 4 (b) 5 (c) 3 (d) 2

More Than One Correct Answer Type

11. The number of divisors of 2014, 2015 and 2022 are in

(a) A.P.

(b) G.P.

(c) Neither A.P. nor G.P.

(d) Can't be determined.

12. If $\lambda(26)^{25} = (26)^{25} + 2\cdot27(26)^{24} + 3\cdot(27)^2(26)^{23} + ... + 26(27)^{25}$, then

(a) $\sqrt{\lambda-100}=24$ (b) $\sqrt{\lambda-51}=25$

(c) $\sqrt{\lambda+53}=27$ (d) $\sqrt{\lambda+165}=29$

13. Let a, b, c, d and e be five numbers such that a, b, c are in A.P. b, c, d are in G.P. and c, d, e are in H.P. if $a = 2$, $e = 72$, then b equals

(a) 5 (b) −7 (c) 7 (d) −5

14. If three distinct numbers $a, b, c \in$ H.P. and $a^2, b^2, c^2 \in$ A.P., then the ratio $a : b : c$ is

(a) $1+\sqrt{3}:-2:1-\sqrt{3}$ (b) $1:\sqrt{3}:-\sqrt{2}$

(c) $1-\sqrt{3}:-2:1+\sqrt{3}$ (d) $1:-\sqrt{3}:\sqrt{2}$

15. If $a, b, c \in$ A.P. and $a^2, b^2, c^2 \in$ H.P., then

(a) a, b, c are in G.P. (b) $-\frac{a}{2}, b, c$ are in G.P.

(c) a, b, c are equal (d) $b^2 = c^2 = \frac{a^2}{2}$

16. If $S_n = 1 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ... \ \forall\ n \in N$, then

(a) sum of even number of terms is negative.

(b) sum of odd number of terms is positive.

(c) $S_{100} = -5050$ (d) $S_{2n+1} < S_{2n-1}$

17. The natural numbers are divided into rows as follows.

1
2 3
4 5 6
7 8 9 10
- - - - - - - - - - - -
- - - - - - - - - - - -

then

(a) number of elements in the n^{th} row is n
(b) 64^{th} row ends with the number 2080
(c) most middle elements in 65^{th} row is 2113
(d) the sum of the elements of 63^{rd} row is 125055

Comprehension Type

Paragraph for Q.No. 18 to 21

Consider the sequence of integers in the form of group (1), (2, 2), (3, 3, 3), ..., (n, n, ..., n) n times

18. If n occurs as 1100 term and the term is divisible by sum of the digits of n, the sum of the digits of n is

(a) 2 (b) 7 (c) 10 (d) 11

19. The sum of first 1100 terms is

(a) 31511 (b) 893 (c) 34404 (d) 35720

20. The sum of the remaining terms in the group after 1100 term in which 1100 term lies is

(a) 1316 (b) 1216 (c) 11316 (d) 11216

21. The most middle term in the group (n, n, ..., n), where n = 47 is

(a) 1104 (b) 1105
(c) 1106 (d) None of these

Matrix Match Type

22. Match the following:

	Column-I		Column-II
P.	$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{r}{\left(\begin{array}{l}\text{product of first } (r+1)\\ \text{positive odd integers}\end{array}\right)}$ is	1.	6
Q.	The sum of infinite series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots$ is	2.	3
R.	The sum of infinite series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....$ is	3.	1/2

Numerical Answer Type

23. If a function f satisfies the condition $f(x + y) = f(x)f(y)$ ∀ all natural number x and y and $f(1) = 3$. Further if k is the natural number and $\sum\limits_{k=1}^{n} f(a+k)=243\frac{(3^n-1)}{2}$ then the value of a equals ______.

24. If 1(2016) + 2(2015) + 3(2014) + ... + 2016(1) = 336 × 2017 × α, then value of α equals ______.

25. The value of n satisfies the equation

$$2 + (6 \times 2^2 - 4\cdot2) + (6 \times 3^2 - 4\cdot3) + ... + (6 \times n^2 - 4n) = 728$$

where n is any natural number, then n equals ______.

26. Different size of sticks are arranged in a row to form an equilateral triangle. The first row consists one stick, the second row consists two sticks and the third row consists three sticks and so on. If 771 sticks are added then all the sticks can be arranged in the shape of a square and each side of the square contains 11 sticks less than the side of the triangle. If the initial number of sticks are $39k$, then k must be equal to ______.

27. If $\sum\limits_{r=1}^{n} t_r=\frac{n(n+1)(n+2)}{4}$, where t_r denotes the r^{th} term of a series and $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{2019}{t_r}=673\lambda$, then λ equals ______.

28. The successive terms of an A.P. are $a_1, a_2, a_3,$ ..... . If $a_3 + a_6 + a_9 + a_{12} = 40$ and $\sum\limits_{r=1}^{14} a_r$ equals to 20λ, then λ equals ______.

SOLUTIONS

1. (a) : Given, $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+....\infty=\frac{\pi^2}{6}$ ...(i)

Now, $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+....\infty$

$$=\frac{\pi^2}{6}-2\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+....\infty\right)$$

$$=\frac{\pi^2}{6}-\frac{2}{4}\left(1+\frac{1}{2^2}+\frac{1}{3^2}+....\infty\right)=\frac{\pi^2}{6}-\frac{1}{2}\times\frac{\pi^2}{6}=\frac{\pi^2}{12}$$

(Using (i))

2. (b) : By using V_n method, we get

$$\text{Required sum}=\frac{1}{2}\left[\frac{1}{1\cdot2}-\frac{1}{11\cdot12}\right]=\frac{1}{4}\left[1-\frac{1}{66}\right]=\frac{65}{264}=\frac{m}{n}$$

(given)

$\therefore$ $m + n = 65 + 264 = 329$

$\therefore$ Sum of digits of $m + n = 14$

3. **(c) :** $\because$ Last term of the 1st group is $1 = 1^2$

Last term of the 2nd group is $4 = 2^2$

Last term of the 3rd group is $9 = 3^2$

……………………………………………

……………………………………………

Last term of the $(n-1)$th group is $(n-1)^2$

Last term of the nth group is n^2 and the first term of this group is obtained by adding 1 in the last term of $(n-1)$th group that is $(n-1)^2 + 1$ *i.e.*, last group is $\{(n-1)^2 + 1, (n-1)^2 + 2, ..., (n-1)^2 + 2n - 1\}$

$\therefore$ Mean value of first and last term of the nth group

$$=\frac{(n-1)^2+1+n^2}{2}=n^2-n+1$$

4. **(b) :** As $2^{2\sin 2x-1}, 3, 2^{4-2\sin 2x} \in$ A.P.

$\therefore$ $6 = 2^{2\sin 2x-1} + 2^{4-2\sin 2x}$

$$\Rightarrow \quad 6=\frac{2^{2\sin 2x}}{2}+\frac{16}{2^{2\sin 2x}}$$

$$\Rightarrow \quad 6=\frac{t}{2}+\frac{16}{t} \qquad \text{(By putting } t=2^{2\sin 2x}\text{)}$$

$\Rightarrow$ $t^2 - 12t + 32 = 0$ $\Rightarrow$ $(t-4)(t-8) = 0$

$\Rightarrow$ $t = 4, 8$, *i.e.*, $2^2, 2^3$

$\therefore$ $2^{2\sin 2x} = 2^2, 2^3$ $\Rightarrow$ $2\sin 2x = 2, 3$

$$\therefore \ \sin 2x=1 \quad \left(\because \ \sin 2x=\frac{3}{2}>1, \text{which is not possible}\right)$$

Now, t_1 = first term = $2^{2-1} = 2$; $t_2 = 3$, $t_3 = 2^2 = 4$

$\therefore$ A.P. is 2, 3, 4, ...

$\Rightarrow$ Common difference, $d = 1$

$\therefore$ $t_{2018} = a + 2017d = 2 + 2017(1) = 2019$

5. **(a) :** $(1-x)(1-3x)(1-3^2x)(1-3^3x)$ ..... $(1-3^{16}x)$

$= (-1)^{17}(x-1)(3x-1)(3^2x-1)(3^3x-1)$ ..... $(3^{16}x-1)$

$$=-3^{1+2+3+...+16}\cdot(x-1)\left(x-\frac{1}{3}\right)\left(x-\frac{1}{3^2}\right)......\left(x-\frac{1}{3^{16}}\right)$$

$$=-3^{\frac{16\times17}{2}}\cdot\left\{x^{17}-\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+....+\frac{1}{3^{16}}\right)x^{16}+....\right\}$$

$\therefore$ Coefficient of x^{16}

$$=-(3^{136})\left\{(-1)\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+....+\frac{1}{3^{16}}\right)\right\}$$

$$=3^{136}\left\{\frac{1\left(1-\left(\frac{1}{3}\right)^{17}\right)}{1-\frac{1}{3}}\right\}=\frac{3^{137}}{2}\left(1-\frac{1}{3^{17}}\right)=\frac{1}{2}3^{120}(3^{17}-1)$$

6. **(b) :** As $f_n(S) = S\,S\,S$ ..... S (n digits)

$$=S\,[111......1\ (n\text{ digits})]=\frac{S}{9}[999.....9\ \ (n\text{ digits})]$$

$$i.e., \ f_n(S)=\frac{S}{9}(10^n-1)$$

Now, putting $S = 6$ and squaring both sides, we get

$$f_n^2(6)=\frac{36}{81}(10^n-1)^2=\frac{4}{9}(10^n-1)^2 \text{ and } f_n(8)=\frac{8}{9}(10^n-1)$$

$$\therefore \ f_n^2(6)+f_n(8)=\frac{4}{9}(10^n-1)^2+\frac{8}{9}(10^n-1)$$

$$=\frac{4}{9}(10^n-1)[10^n-1+2]=\frac{4}{9}(10^n-1)(10^n+1)$$

$$=\frac{4}{9}(10^{2n}-1)=4\left[\frac{1}{9}(10^{2n}-1)\right]=4\cdot f_{2n}(1)$$

7. **(a) :** Let $2, a_1, a_2, a_3, a_4, 12 \in$ A.P. ....(i)

$2, g_1, g_2, g_3, g_4, 12 \in$ G.P. ....(ii)

$2, h_1, h_2, h_3, h_4, 12 \in$ H.P. ....(iii)

From (i), we have

$t_6 = 12 = 2 + 5d$ $\Rightarrow$ $d = 2$

$\therefore$ Terms of A.P. are 2, 4, 6, 8, 10, 12

From (ii), we have

$t_6 = 12 = 2r^5 \Rightarrow r = (6)^{1/5}$

$\therefore$ Terms of G.P. are $2, 2\cdot6^{1/5}, 2\cdot6^{2/5}, 2\cdot6^{3/5}, 2\cdot6^{4/5}, 2\cdot6^{5/5}$

Also, $2, h_1, h_2, h_3, h_4, 12 \in$ H.P.

$\therefore$ Terms of H.P. are

$$2,\frac{(n+1)\times a\times b}{a+nb},\frac{(n+1)\times a\times b}{2a+(n-1)b},\frac{(n+1)\times a\times b}{3a+(n-2)b},$$

$$\frac{(n+1)\times a\times b}{4a+(n-3)b},12$$

$$i.e., 2,\frac{5\times2\times12}{50},\frac{5\times2\times12}{40},\frac{5\times2\times12}{30},\frac{5\times2\times12}{20},12$$

$[\because \ n = 4, a = 2, b = 12]$

$$i.e., \ 2,\frac{12}{5},3,4,6,12$$

$\therefore$ First A.M. between 2 and 12 = 4

Second G.M. between 2 and 12 = $2\cdot6^{2/5}$

Third H.M. between 2 and 12 = 4

Which are roots of the required equation.

Now, sum of roots = $8 + 2\cdot6^{2/5}$

Product of roots taken two at a time

$= 16 + 16\cdot6^{2/5} = 16(1 + 6^{2/5})$

Product of roots taken all at a time

$= 16 \cdot 2 \cdot 6^{2/5} = 32\cdot6^{2/5}$

$\therefore$ Required equation is

$x^3 - (8 + 2\cdot6^{2/5})x^2 + 16(1 + 6^{2/5})x - 32\cdot6^{2/5} = 0$

8. **(a) :** Let a be the first term and r be the common ratio of the given G.P. then

$$\alpha=\sum_{n=1}^{2019} t_{2n}=t_2+t_4+t_6+.....+t_{4038}$$

$= ar + ar^3 + ar^5 + ... + ar^{4037}$

∴ $\alpha = ar(1 + r^2 + r^4 + ... + r^{4036})$ ....(i)

and, $\beta=\sum_{n=1}^{2019} t_{2n-1}=t_1+t_3+t_5+.....+t_{4037}$

$= a + ar^2 + ar^4 + ... + ar^{4036}$

∴ $\beta = a(1 + r^2 + r^4 + ... + r^{4036})$ ....(ii)

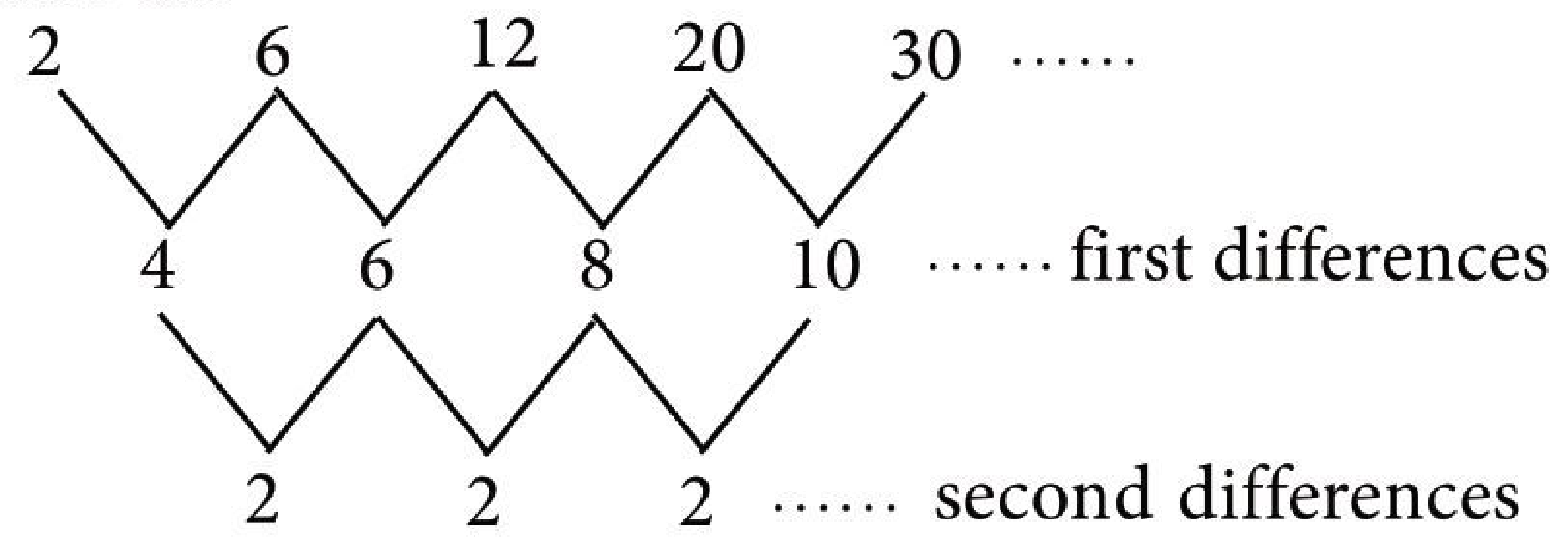
On dividing (i) and (ii), we have, $\dfrac{\alpha}{\beta}=r$

9. (c) : Let $S = \dfrac{1\cdot 2}{3}+\dfrac{2\cdot 3}{3^2}+\dfrac{3\cdot 4}{3^3}+.......\infty$

∴ $t_n = n(n+1)\cdot x^n$, where $x=\dfrac{1}{3}$

Now, $S = 2x + 6x^2 + 12x^3 + 20x^4 + ... \infty$

The terms in the successive order of differences of the series are



The second order differences are equal hence the scale of relation shall be $(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

$$\left.\begin{array}{lrrrrl} \text{Now, } S = & 2x + & 6x^2 + & 12x^3 + & 20x^4 + & 30x^5 + ... \\ \therefore\ -3xS = & & -6x^2 - & 18x^3 - & 36x^4 - & 60x^5 + ... \\ 3x^2S = & & & 6x^3 + & 18x^4 + & 36x^5 + ... \\ -x^3S = & & & & -2x^4 - & 6x^5 - .... \end{array}\right]\ (*)$$

Adding the terms of (*) column wise, we get

$S(1 - x)^3 = 2x$ (All other terms get cancelled)

∴ $S=\dfrac{2x}{(1-x)^3}$; putting $x=\dfrac{1}{3}$, we get $S=\dfrac{9}{4}$

10. (b) : Let the decreasing geometric series be a, ar, ar^2, ar^3, ...

Now, $\dfrac{t_{10}}{t_8}=\dfrac{ar^9}{ar^7}=\dfrac{1}{25} \Rightarrow r^2=\dfrac{1}{25} \Rightarrow r=\pm\dfrac{1}{5}$

$\Rightarrow r=\dfrac{1}{5}$ (As series is decreasing geometric series and $r=-\dfrac{1}{5}$ is not possible)

Again, $\dfrac{t_3}{t_2^2}=\dfrac{ar^2}{(ar)^2}=\dfrac{1}{64} \Rightarrow \dfrac{1}{a}=\dfrac{1}{64} \Rightarrow a=64$

∴ $S_\infty=\dfrac{a}{1-r}=\dfrac{64}{1-\dfrac{1}{5}}=\dfrac{64\times 5}{4}=80=16k \Rightarrow k=5$

11. (a, b) : We have, 2014 = 2 × 19 × 53

∴ Number of divisors = (1 + 1)(1 + 1)(1 + 1) = 8

2015 = 5 × 13 × 31

∴ Number of divisors = (1 + 1)(1 + 1)(1 + 1) = 8

2022 = 2 × 3 × 337

∴ Number of divisors = (1 + 1)(1 + 1)(1 + 1) = 8

The number of divisors of 2014, 2015 and 2022 are 8, 8 and 8 respectively, which are in A.P. as well as in G.P.

12. (a, b, c, d) : We have $\lambda(26)^{25} = (26)^{25} + 2\cdot 27(26)^{24} + 3\cdot(27)^2(26)^{23} + ... + 26(27)^{25}$

$\Rightarrow \lambda=1+2\cdot\left(\dfrac{27}{26}\right)^1+3\left(\dfrac{27}{26}\right)^2+......+26\left(\dfrac{27}{26}\right)^{25}$

$\Rightarrow \lambda=1+2x+3x^2+....+26x^{25}\left(\text{where } x=\dfrac{27}{26}\right)$ ...(i)

∴ $\lambda x = x + 2x^2 + ... + 25x^{25} + 26x^{26}$ ...(ii)

On subtracting (ii) from (i), we get

$\lambda(1 - x) = 1 + x + x^2 + x^3 + ... + x^{25} - 26x^{26}$

$\Rightarrow \lambda(1-x)=\dfrac{x^{26}-1}{x-1}-26x^{26}$

$\Rightarrow \lambda(1-x)=\dfrac{x^{26}-1-26x^{27}+26x^{26}}{x-1}$

$\Rightarrow \lambda(1-x)=\dfrac{27x^{26}-26x^{27}-1}{x-1}$

$\Rightarrow \lambda=\dfrac{1}{(x-1)^2}$ $\left(\because\ 27x^{26}=26x^{27} \text{ as } x=\dfrac{27}{26}\right)$

∴ $\lambda = (26)^2 = 676$

∴ $\sqrt{\lambda-100}=\sqrt{576}=24, \sqrt{\lambda-51}=\sqrt{625}=25,$

$\sqrt{\lambda+53}=\sqrt{729}=27$ and $\sqrt{\lambda+165}=\sqrt{841}=29$

13. (c, d) : ∵ $a, b, c \in$ A.P. $\Rightarrow 2b = a + c$

$\Rightarrow b=\dfrac{2+c}{2}$ $(\because\ a = 2)$

∴ $b, c, d \in$ G.P. $\Rightarrow c^2 = bd$

and $c, d, e \in$ H.P.

$\Rightarrow d=\dfrac{2ce}{c+e}=\dfrac{2c\times 72}{c+72}$ $(\because\ e = 72)$

Now, $bd = c^2$

$\Rightarrow \dfrac{(2+c)}{2}\cdot\dfrac{(2c\times 72)}{(c+72)}=c^2 \Rightarrow 72(2+c)=c(c+72)$

$\Rightarrow c^2 = 144 \Rightarrow c = \pm 12$

Now, $b=\dfrac{2+c}{2} \Rightarrow b=7,-5$

14. (a, c) : We have, $a \neq b \neq c$ and $a, b, c \in$ H.P.

$\Rightarrow \quad b=\frac{2ac}{a+c} \quad \Rightarrow \quad \frac{b}{2}=\frac{ac}{a+c}=k$ (say)

$\Rightarrow \quad b = 2k,\ ac = k(a + c)$ ....(*)

Again, $a^2, b^2, c^2 \in$ A.P.

$\Rightarrow \quad 2b^2 = a^2 + c^2 = (a + c)^2 - 2ac$

$\Rightarrow \quad 8k^2 = (a + c)^2 - 2k(a + c)$ [Using (*)]

$\Rightarrow \quad (a + c)^2 - 2(a + c)k - 8k^2 = 0$

$\Rightarrow \quad (\overline{a+c}-4k)(\overline{a+c}+2k)=0 \quad \Rightarrow \quad a + c = 4k, -2k$

When $a + c = 4k$ then $ac = 4k^2$ and when $a + c = -2k$ then $ac = -2k^2$

Now, $(a - c)^2 = (a + c)^2 - 4ac = 16k^2 - 16k^2 = 0$

$\Rightarrow \quad a = c$ which contradicts the hypothesis

$\therefore \quad a + c = -2k$ and $ac = -2k^2$

Now, $(a - c)^2 = (a + c)^2 - 4ac = 4k^2 + 8k^2 = 12k^2$

$\Rightarrow \quad a-c=\pm 2\sqrt{3}k$

Now, $a+c=-2k$ and $a-c=\pm 2\sqrt{3}k$

$\therefore \quad a=(-1\pm\sqrt{3})k$ and $c=(-1\mp\sqrt{3})k$

$\therefore \quad a:b:c=-1\pm\sqrt{3}\ :2:-1\mp\sqrt{3}$

or $\quad a:b:c=1\mp\sqrt{3}:-2:1\pm\sqrt{3}$

$\therefore \quad a:b:c=(1-\sqrt{3}:-2:1+\sqrt{3})$

or $\quad a:b:c=1+\sqrt{3}:-2:1-\sqrt{3}$

15. (a, b, c) : $\because a, b, c \in$ A.P. $\Rightarrow \quad 2b = a + c$ ... (i)

Again, $a^2, b^2, c^2 \in$ H.P. $\Rightarrow \ b^2=\frac{2a^2c^2}{a^2+c^2}$ ... (ii)

From (i) and (ii), we have, $\frac{(a+c)^2}{4}=\frac{2a^2c^2}{(c^2+a^2)}$

$\Rightarrow \quad (a + c)^2(c^2 + a^2) - 8a^2c^2 = 0$

$\Rightarrow \quad (a^2 + c^2)^2 + 2ac(a^2 + c^2) - 8a^2c^2 = 0$

$\Rightarrow \quad (a^2 + c^2 + 4ac)(a^2 + c^2 - 2ac) = 0$

$\Rightarrow \quad a^2 + c^2 + 4ac = 0$ or $a^2 + c^2 - 2ac = 0$

$\Rightarrow \quad (a + c)^2+ 2ac = 0$ or $(a - c)^2 = 0 \Rightarrow a = c$

$\Rightarrow \quad (2b)^2 + 2ac = 0$ or $2b = 2a = 2c \quad (\because 2b = a + c)$

$\Rightarrow \quad 4b^2 = -2ac$ or $a = b = c$

$\Rightarrow \quad b^2=\frac{-ac}{2}$ or a, b, c are both in A.P. and G.P.

$\Rightarrow \quad b^2=\left(-\frac{a}{2}\right)(c) \quad \Rightarrow \quad -\frac{a}{2}, b, c$ are in G.P.

16. (a, b, c) : S_2 = Sum of first two terms *i.e.*, sum of even number of terms

$= 1 - 2^2 = 1 - 4 = -3 < 0 \quad \therefore$ Option (a) is correct.

S_3 = Sum of first three terms *i.e.*, sum of odd number of terms

$= t_1 + t_2 + t_3 = 1 - 2^2 + 3^2 = 1 - 4 + 9 = 6 > 0$

$\therefore$ Option (b) is correct.

Also, $S_{100} = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots + (99^2 - 100^2)$

$= (-1)[(1 + 2) + (3 + 4) + (5 + 6) + \ldots + (99 + 100)]$

$= (-1)[1 + 2 + 3 + 4 + \ldots + 100]$

$=(-1)\frac{100\times 101}{2}=-5050 \quad \therefore$ Option (c) is correct.

Now, S_{2n+1} and S_{2n-1} both represent the sum of odd terms and $S_1 < S_3$, $S_3 < S_5$.

Therefore, in general $S_{2n-1} < S_{2n+1}$

Thus, option (d) is not correct.

17. (a, b, c, d) : The number of elements in first, second, third row are 1, 2, 3 respectively therefore n^{th} row consists of n elements. So, option (a) is correct.

The rows are ending with numbers 1, 3, 6, 10, ...

5th row ends with $1 + 2 + 3 + 4 + 5 = \frac{5\times 6}{2}=15$

$\therefore \quad n^{\text{th}}$ row ends with $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$\therefore \quad 64^{\text{th}}$ row ends with $\frac{64\times 65}{2}=(32\times 65) = 2080$

Thus, option (b) is correct.

Again as 65th row having 65 elements and 33rd element is the most middle element in this row is obtained by adding 33 to the last element of 64th row, which is 2080

$\therefore$ Most middle element in 65th row = 2080 + 33 = 2113 (option (c))

Now for sum of the element of 63rd row, we need either the last element of 62nd row or the first element of 63rd row. The last element of 62nd row = $\frac{62\times 63}{2}=1953$

$\therefore$ First element of 63rd row is 1954 and last element of 63rd row = $\frac{63\times 64}{2} = 2016$

$\therefore$ Sum of the elements of 63rd row

$=\frac{n}{2}(a+l)=\frac{63}{2}(1954+2016)=125055$ (Option (d))

(18-21): Deduction from the sequence of integers in the form of groups

$$(1), (2, 2), (3, 3, 3), \ldots, \underbrace{(n, n, n, \ldots, n)}_{n\text{-times}}$$

- The number of digits in 1st, 2nd, 3rd, ..., n^{th} group are respectively 1, 2, 3, 4, ..., n.
- Sum of the digits of 1st, 2nd, 3rd, ..., n^{th} group are $1^2, 2^2, 3^2, \ldots, n^2$. Therefore sum of the terms upto n^{th} group is $1^2 + 2^2 + 3^2 + \ldots + n^2$.
- Sum of the different terms upto n^{th} group is $\frac{n(n+1)}{2}$, which also represent the total number of terms upto n^{th} group.

- In each group each digit is same in single digit number and in two digits it occurs in identical pair.
- The first term of $(n+1)^{\text{th}}$ group is $\left(\dfrac{n(n+1)}{2}+1\right)^{\text{th}}$ term.

18. (d) : Now, let 1100 term falls in n^{th} group, then

$$\frac{n(n-1)}{2} < 1100 \le \frac{n(n+1)}{2}$$

$\Rightarrow$ $n(n-1) < 2200$ and $n(n+1) \ge 2200$

$\Rightarrow$ $47 \times 46 = 2162 < 2200$ and $47 \times 48 = 2256 > 2200$

$\Rightarrow$ $n = 47$

$\therefore$ Sum of the digits of $n = 4 + 7 = 11$

19. (c) : Again total terms upto 46th group is

$$\frac{46\times 47}{2} = 1081$$

$\therefore$ Sum of first 1100 terms

$= 1^2 + 2^2 + \ldots + 46^2 + (1100 - 1081) \times 47$

$$= \frac{46\times 47\times 93}{6} + 47\times 19 = 33511 + 893 = 34404$$

20. (a) : As upto 47th group the total number of terms

$$= \frac{47\times 48}{2} = 1128 \text{ (each having value 47)}$$

$\therefore$ Terms left for counting after 1100th term

$= 1128 - 1100 = 28$

$\therefore$ Required sum of remaining 28 terms

$= 47 \times 28 = 1316$

21. (b) : Now, the first n occurs as 1082th term and the last n occurs as 1128th term in the 47th group and 24th forty seven is the most middle item in this group.

$\therefore$ Most middle term is $(1128 - 23)^{\text{th}}$ or $(1081 + 24)^{\text{th}}$

$= 1105^{\text{th}}$ term

22. (P) → (3), (Q) → (1), (R) → (2)

(P) We have,

$$t_r = \frac{r}{1\cdot 3\cdot 5\ldots\cdot(2r+1)} = \frac{1}{2}\left[\frac{2r+1-1}{1\cdot 3\cdot 5\ldots(2r+1)}\right]$$

$$\therefore\ t_r = \frac{1}{2}\left[\frac{1}{1\cdot 3\cdot 5\ldots(2r-1)} - \frac{1}{1\cdot 3\cdot 5\ldots(2r+1)}\right]$$

$$= -\frac{1}{2}[V_n - V_{n-1}] \qquad \text{(Using } V_n \text{ method)}$$

$$\therefore\ \sum_{r=1}^{n} t_r = -\frac{1}{2}[V_n - V_0] = \frac{1}{2}\left(1 - \frac{1}{1\cdot 3\cdot 5\ldots(2n+1)}\right)$$

$$\therefore\ \lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{1}{2}\left(1 - \frac{1}{1\cdot 3\cdot 5\ldots(2n+1)}\right) = \frac{1}{2}$$

(Q) Let $S_r = \dfrac{3}{1^2} + \dfrac{5}{1^2+2^2} + \dfrac{7}{1^2+2^2+3^2} + \ldots + \dfrac{2r+1}{1^2+2^2+\ldots+r^2}$

$$\therefore\ t_r = \frac{2r+1}{1^2+2^2+\ldots+r^2} = \frac{6(2r+1)}{r(r+1)(2r+1)}$$

$$= \frac{6}{r(r+1)} = 6\left[\frac{1}{r} - \frac{1}{r+1}\right]$$

$$\therefore\ \sum_{r=1}^{n} t_r = 6\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) \quad \therefore\ S_n = 6\left[1 - \frac{1}{n+1}\right]$$

$\therefore$ $S_\infty = 6(1 - 0) = 6$

(R) Let $S = 1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4} + \ldots \infty$ ... (i)

$\therefore$ $\dfrac{1}{3}S = \dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{6}{3^3} + \dfrac{10}{3^4} + \ldots \infty$ ... (ii)

Subtracting (ii) from (i), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots \infty$$

$$= \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots \infty$$

$$= \frac{4}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \infty\right) = \frac{4}{3} \times \frac{1}{\left(1 - \frac{1}{3}\right)} = \frac{4}{3} \times \frac{3}{2} = 2$$

$$\therefore\ S = \frac{2\times 3}{2} = 3$$

23. (4) : We have, $f(1) = 3$ and $f(x+y) = f(x)\cdot f(y)$.

Putting $x = 1$, $y = 1$, we get

$f(1+1) = f(1)\,f(1) = [f(1)]^2 \Rightarrow f(2) = 3^2$

Also, $f(1+2) = f(1)\,f(2) \Rightarrow f(3) = 3^3$

And $f(1+3) = f(1)\,f(3) = 3\cdot 3^3 = 3^4 \Rightarrow f(4) = 3^4$

....

....

Similarly, $f(k) = 3^k$ and $f(a) = 3^a$ .... (*)

$$\text{Now, } \sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)\cdot f(k)$$

$$= f(a)\sum_{k=1}^{n} f(k) = f(a)\sum_{k=1}^{n} 3^k$$

$= f(a)[3^1 + 3^2 + 3^3 + \ldots + 3^n]$

$$\therefore\ \sum_{k=1}^{n} f(a+k) = 3f(a)\frac{(3^n - 1)}{2}$$

$$\Rightarrow\ 243\frac{(3^n-1)}{2} = 3f(a)\left(\frac{3^n-1}{2}\right)$$

$\Rightarrow$ $81 = f(a) = 3^a$

$\Rightarrow$ $3^4 = 3^a \Rightarrow a = 4$

24. (2018) : $\because$ 1(2016) + 2(2015) + 3(2014) + ... + 2016(1)

$$= \sum_{r=1}^{2016} r(2016-(r-1)) = \sum_{r=1}^{2016} r(2017-r)$$

$$= \sum_{r=1}^{2016} 2017r - \sum_{r=1}^{2016} r^2$$

$$= \frac{2017 \times 2016 \times 2017}{2} - \frac{2016 \times 2017 \times 4033}{6}$$

$$= \frac{2017 \times 2016}{6}(6051 - 4033) = 2017 \times 336 \times 2018$$

$$= 2017 \times 336 \times \alpha \Rightarrow \alpha = 2018$$

25. (7) : Let $S = 2 + (6 \times 2^2 - 4 \cdot 2) + (6 \times 3^2 - 4 \cdot 3) + ... + (6 \times n^2 - 4n)$

$$\therefore \quad S = 2 + 6(2^2 + 3^2 + ... + n^2) - 4(2 + 3 + 4 + ... + n)$$

$$= 2 + 6(1^2 + 2^2 + ... + n^2) - 4(1 + 2 + 3 + ... + n) - 6 + 4$$

$$= 6(1^2 + 2^2 + ... + n^2) - 4(1 + 2 + 3 + ... + n)$$

$$= 6 \cdot \frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2}$$

$$= n(n + 1)[2n + 1 - 2] = n(n + 1)(2n - 1)$$

$$\therefore \quad n(n+1)(2n - 1) = 728 = 7 \times 8 \times 13$$

$$= 7(7 + 1)(2 \times 7 - 1) \Rightarrow n = 7$$

26. (55) : As the first row contains 1 stick, 2nd row contains 2 sticks and third row contains 3 sticks, and so on.

$\therefore$ Initially the number of sticks = 1 + 2 + 3 + ... + n

$$= \frac{n(n+1)}{2} \qquad \text{... (i)}$$

According to problem, we have

$$\frac{n(n+1)}{2} + 771 = (n-11)^2$$

$$\Rightarrow \frac{n^2+n+1542}{2} = n^2 - 22n + 121$$

$$\Rightarrow n^2 + n + 1542 = 2n^2 - 44n + 242$$

$$\Rightarrow n^2 - 45n - 1300 = 0$$

$$\Rightarrow (n - 65)(n + 20) = 0 \Rightarrow n = 65 \quad [\because n \neq -20]$$

$\therefore$ Initially the number of sticks

$$= \frac{n(n+1)}{2} = \frac{65 \times 66}{2} = 2145$$

$$\because \quad 39k = 2145 \Rightarrow k = 55$$

27. (4) : We have, $\displaystyle\sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)}{4} = S_n(\text{say})$

$$\therefore \quad \sum_{r=1}^{n-1} t_r = \frac{(n-1)(n)(n+1)}{4} = S_{n-1}$$

$$\therefore \quad t_n = \frac{n(n+1)(n+2)}{4} - \frac{(n-1)(n)(n+1)}{4}$$

$$= \frac{n(n+1)}{4}[n+2-n+1] = \frac{3n(n+1)}{4}$$

$$\therefore \quad \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{t_r} = \frac{4}{3} \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{r(r+1)}$$

$$= \frac{4}{3} \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{r} - \frac{1}{r+1}\right) \qquad \text{(Using partial fraction)}$$

$$= \frac{4}{3} \lim_{n\to\infty} \left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + ... + \left(\frac{1}{n-1}-\frac{1}{n}\right) + \left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$= \frac{4}{3} \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = \frac{4}{3}(1-0) = \frac{4}{3}$$

$$\therefore \quad \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2019}{t_r} = \frac{4}{3} \times 2019 = 673\lambda \Rightarrow \lambda = 4$$

28. (7) : Let the common difference of A.P. is d and $a_1 = a$ be the first term, then series becomes a, $a + d$, $a + 2d$, ...

Now, $a_3 + a_6 + a_9 + a_{12} = 40$

$$\Rightarrow (a + 2d) + (a + 5d) + (a + 8d) + (a + 11d) = 40$$

$$\Rightarrow 4a + 26d = 40 \Rightarrow 2a + 13d = 20$$

$$\therefore \quad \sum_{r=1}^{14} a_r = S_{14} = \frac{14}{2}[2a + 13d] = 7 \times 20 = 140$$

$$\because \quad \sum_{r=1}^{14} a_r = 20\lambda \Rightarrow 140 = 20\lambda \Rightarrow \lambda = 7$$

◆ ◆

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SECTION-I

Single Option Correct Type

1. If $\int \sec^{4/3}x \operatorname{cosec}^{8/3}x\,dx = a(\tan x)^{-5/3} + b(\tan x)^{1/3} + C$, then $5a + b =$

(a) 3 (b) -3 (c) 0 (d) -1

2. A parallelepiped is formed by planes drawn parallel to the coordinate axes through the points $A = (2, 3, 4)$ and $B = (11, 9, 8)$. Then, the volume of the parallelepiped is equal to (in cubic units)

(a) 108 (b) 36 (c) 72 (d) 216

3. In a triangle ABC, if r_1, r_2, r_3 are the roots of the equation $x^3 + \alpha x^2 + \beta x + v = 0$, then $\beta =$

(a) r^2 (b) rs (c) s^2 (d) rR

4. If 1, α_1, α_2, α_3 and α_4 are the fifth roots of unity, then $\sum_{r=1}^{4}\frac{1}{2-\alpha_r} =$

(a) $\frac{51}{31}$ (b) $\frac{49}{31}$ (c) $\frac{25}{32}$ (d) $\frac{25}{16}$

5. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$, then the value of $\frac{a_7}{a_{13}}$, is

(a) 6 (b) 8 (c) 12 (d) 16

6. Let the sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ form an A.P. Then, $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2$ is equal to

(a) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (b) $\frac{2n}{2n-1}(a_{2n}^2 - a_1^2)$

(c) $\frac{n}{n+1}(a_1^2 - a_{2n}^2)$ (d) None of these

7. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to 9/10, then n is greater than

(a) $\frac{1}{\log_{10}4 - \log_{10}3}$ (b) $\frac{1}{\log_{10}4 + \log_{10}3}$

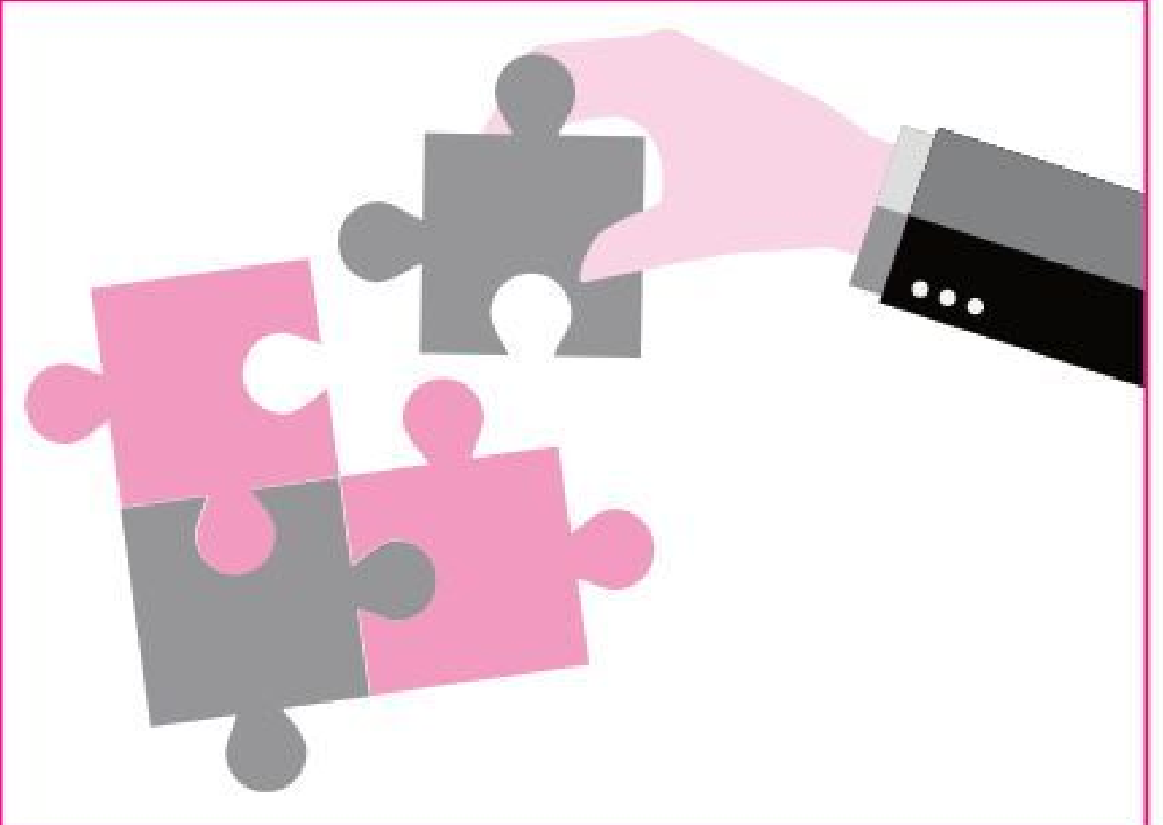
(c) $\frac{9}{\log_{10}4 - \log_{10}3}$ (d) $\frac{4}{\log_{10}4 - \log_{10}3}$

8. Tangents to the hyperbola $x^2 - 9y^2 = 9$ are drawn from (3, 2), then

(a) equation of tangent is $x = 3$

(b) equation of tangent is $5x - 12y + 9 = 0$

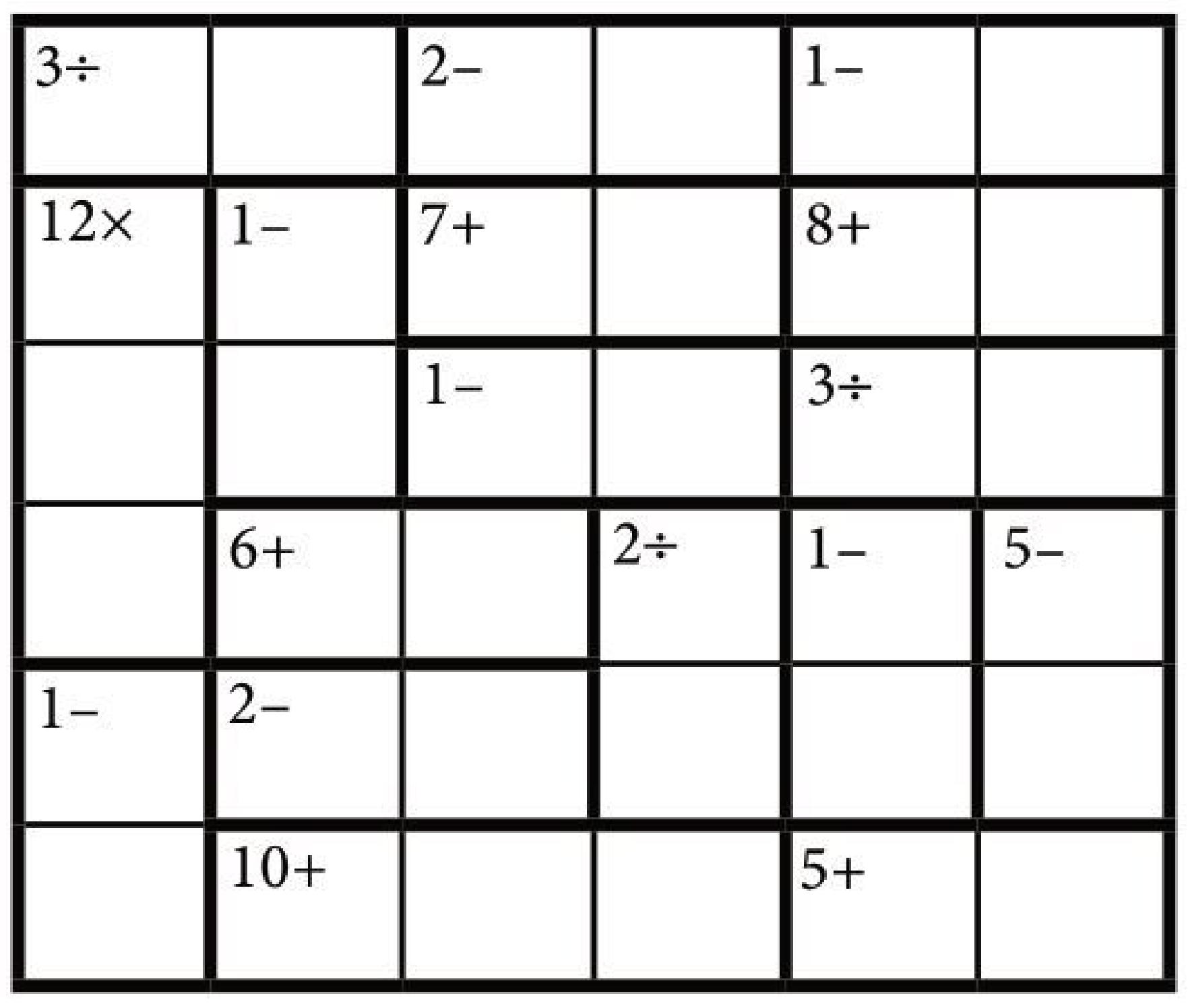
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3÷		2–		1–	
12×	1–	7+		8+	
		1–		3÷	
	6+		2÷	1–	5–
1–	2–				
	10+			5+	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

(c) the area of the triangle that these tangents form with their chord of contact is 8 sq. units.

(d) All of these

9. Form the differential equation corresponding to $y = 2(x^2 - 1) + ce^{-x^2}$.

(a) $\dfrac{dy}{dx} = 4x^3 + xy$ (b) $\dfrac{dy}{dx} = 4x^3 + 2xy$

(c) $\dfrac{dy}{dx} = 4x^2 + 2xy$ (d) $\dfrac{dy}{dx} = 4x^3 - 2xy$

10. The area of the region bounded by the curves $y = x^3$, $y = \dfrac{1}{x}$ and $x = 2$ is

(a) $4 - \log_e 2$ sq. units (b) $\dfrac{1}{4} + \log_e 2$ sq. units

(c) $3 - \log_e 2$ sq. units (d) $\dfrac{15}{4} - \log_e 2$ sq. units

SECTION-II

Numerical Answer Type

11. Let $\vec{a}$ is vector whose magnitude is unity and coplanar with the vectors $\vec{b}$ and $\vec{c}$, where $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \hat{k}$ such that $\vec{a}$ is perpendicular to $\vec{b}$. If λ is the projection of $\vec{a}$ along $\vec{c}$, then the value of $\dfrac{11\sqrt{2}\lambda}{4}$ is ________.

12. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then $\dfrac{[\vec{a}+2\vec{b}\ \vec{b}+2\vec{c}\ \vec{c}+2\vec{a}]}{[\vec{a}\,\vec{b}\,\vec{c}]} =$ ________.

13. The equation of the plane which has the property that the point $Q(5, 4, 5)$ is the reflection of point $P(1, 2, 3)$ through that plane, is $ax + by + cz = d$ where $a, b, c, d \in N$. Find the least value of $(a + b + c + d)$.

14. Boxes B_1, B_2 and B_3 contain different coloured balls as given in table.

	White	Black	Red
B_1	2	n	2
B_2	3	2	4
B_3	4	3	2

The probabilities of selecting boxes are, respectively, $\dfrac{1}{6}, \dfrac{1}{2}$ and $\dfrac{1}{3}$. One of the boxes is chosen at random and a ball is drawn from it. If the probability of the drawn ball is black is $\dfrac{23}{90}$, then the value of n is equal to ________.

15. If $I = \displaystyle\int_{-1}^{1} \frac{e^{|x|}}{1+a^x}\,dx$, then the value of $\dfrac{99}{(e-1)} I$ must be equal to ________.

SOLUTIONS

1. (c) : Let $I = \int \sec^{4/3} x \operatorname{cosec}^{8/3} x\, dx$

$$= \int \frac{1}{\cos^{4/3} x \sin^{8/3} x}\,dx = \int \frac{\sec^4 x}{\tan^{8/3} x}\,dx$$

(Dividing numerator and denominator by $\cos^4 x$)

$$= \int (1+\tan^2 x)(\tan x)^{-8/3}\, d(\tan x)$$

$$= \int \{(\tan x)^{-8/3} + (\tan x)^{-2/3}\}\, d(\tan x)$$

$$= -\frac{3}{5}(\tan x)^{-5/3} + 3(\tan x)^{1/3} + C$$

$\therefore$ $a = -\dfrac{3}{5}$ and $b = 3 \Rightarrow 5a + b = 0$

2. (d) : Translating the axes through $A(2, 3, 4)$ then, new position of A is $(0, 0, 0)$ and B is $(9, 6, 4)$

$\therefore$ Coterminous edges are of lengths 9, 6, 4 respectively

$\therefore$ Volume of parallelepiped = 9 × 6 × 4 = 216 cu. units

3. (c) : $\beta = r_1 r_2 + r_2 r_3 + r_3 r_1$

$$= \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)}$$

$$= \frac{\Delta^2 (s-c+s-a+s-b)}{(s-a)(s-b)(s-c)}$$

$= s \cdot (3s - a - b - c) = s \cdot s = s^2$

4. (b) : $z^5 - 1 = (z - 1)(z - \alpha_1)(z - \alpha_2)(z - \alpha_3)(z - \alpha_4)$

$\Rightarrow \ln(z^5 - 1) = \ln(z - 1) + \ln(z - \alpha_1) + \ln(z - \alpha_2) +$

$\Rightarrow \ln(z - \alpha_3) + \ln(z - \alpha_4)$

Differentiating, we get

$$\frac{5z^4}{z^5-1} = \frac{1}{z-1} + \frac{1}{z-\alpha_1} + \frac{1}{z-\alpha_2} + \frac{1}{z-\alpha_3} + \frac{1}{z-\alpha_4}$$

Putting $z = 2$, $\displaystyle\sum_{r=1}^{4} \frac{1}{2-\alpha_r} = \frac{80}{31} - 1 = \frac{49}{31}$

5. (b) : Given, $(2x^2 + 3x + 4)^{10} = \displaystyle\sum_{r=0}^{20} a_r x^r$

Replacing x by $\dfrac{2}{x}$, we get

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x}\right)^r$$

$$\Rightarrow\ 2^{10}(2x^2+3x+4)^{10}=\sum_{r=0}^{20} a_r 2^r x^{20-r}$$

$$\Rightarrow\ \sum_{r=0}^{20} a_r x^r=\sum_{r=0}^{20} a_r 2^{r-10} x^{20-r}$$

Comparing coefficient of x^7 on both sides, we get $a_7 = a_{13} \times 2^3$.

6. (a) : Let d be the common difference of the A.P. Then,

$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_{2n} - a_{2n-1}$.

Now, $a_1{}^2 - a_2{}^2 + a_3{}^2 - a_4{}^2 + \ldots + a_{2n-1}^2 - a_{2n}^2$

$= (a_1 + a_2)(a_1 - a_2) + (a_3 + a_4)(a_3 - a_4) + \ldots + (a_{2n-1} + a_{2n})(a_{2n-1} - a_{2n})$

$= -d(a_1 + a_2 + a_3 + \ldots + a_{2n})$

$$=-dn(a_1+a_{2n})=-dn\frac{(a_1^2-a_{2n}^2)}{a_1-a_{2n}}$$

$$=\frac{dn(a_1^2-a_{2n}^2)}{a_{2n}-a_1}=\frac{dn(a_1^2-a_{2n}^2)}{(2n-1)d}=\frac{n(a_1^2-a_{2n}^2)}{(2n-1)}$$

7. (a) : $\because$ Probability of no success is less than $\frac{1}{10}$.

$$\therefore\ (1-p)^n<\frac{1}{10}\Rightarrow\left(\frac{3}{4}\right)^n<\frac{1}{10}$$

$$\Rightarrow\ \left(\frac{4}{3}\right)^n>10\Rightarrow n>\log_{4/3}10=\frac{1}{\log_{10}(4/3)}$$

$$\therefore\ n>\frac{1}{\log_{10}4-\log_{10}3}.$$

8. (d) : Given equation of hyperbola is $x^2 - 9y^2 = 9$

or $\frac{x^2}{9}-\frac{y^2}{1}=1$

Let the equation of tangent is $y=mx\pm\sqrt{a^2m^2-b^2}$

It passes through (3, 2).

$\therefore\ 2=3m\pm\sqrt{9m^2-1}$

$\Rightarrow$ $4 + 9m^2 - 12m = 9m^2 - 1$ or $m_1 = 5/12$ and $m_2 = \infty$

Equation of tangents are $y - 2 = (5/12)(x - 3)$

$\Rightarrow$ $5x - 12y + 9 = 0$ ...(i) and $x = 3$ ...(ii)

Now equation of chord of contact w.r.t. point $P(3, 2)$ is $T = 0$, *i.e.*, $3x - 18y = 9$ or $x - 6y = 3$ ...(iii)

Solving (i) and (iii), $x = -5$, $y=-\frac{4}{3}$

Solving (ii) and (iii), $x = 3$, $y = 0$

Thus vertices of triangle are (3, 2), (3, 0), (–5, –4/3).

$$\therefore\ \text{Required area}=\frac{1}{2}\begin{vmatrix}3 & 2 & 1\\ 3 & 0 & 1\\ -5 & -\frac{4}{3} & 1\end{vmatrix}$$

$$=\frac{1}{2}\times\left|3\left(\frac{4}{3}\right)-2(3+5)+1(-4)\right|$$

$$=\frac{1}{2}|4-16-4|=8\text{ sq. units}$$

9. (d) : We have, $y = 2(x^2 - 1) + ce^{-x^2}$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\frac{dy}{dx}=4x+ce^{-x^2}(-2x)\ \Rightarrow\ ce^{-x^2}=\frac{\left(\frac{dy}{dx}-4x\right)}{(-2x)}$$

Putting this value in equation (i), we get

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

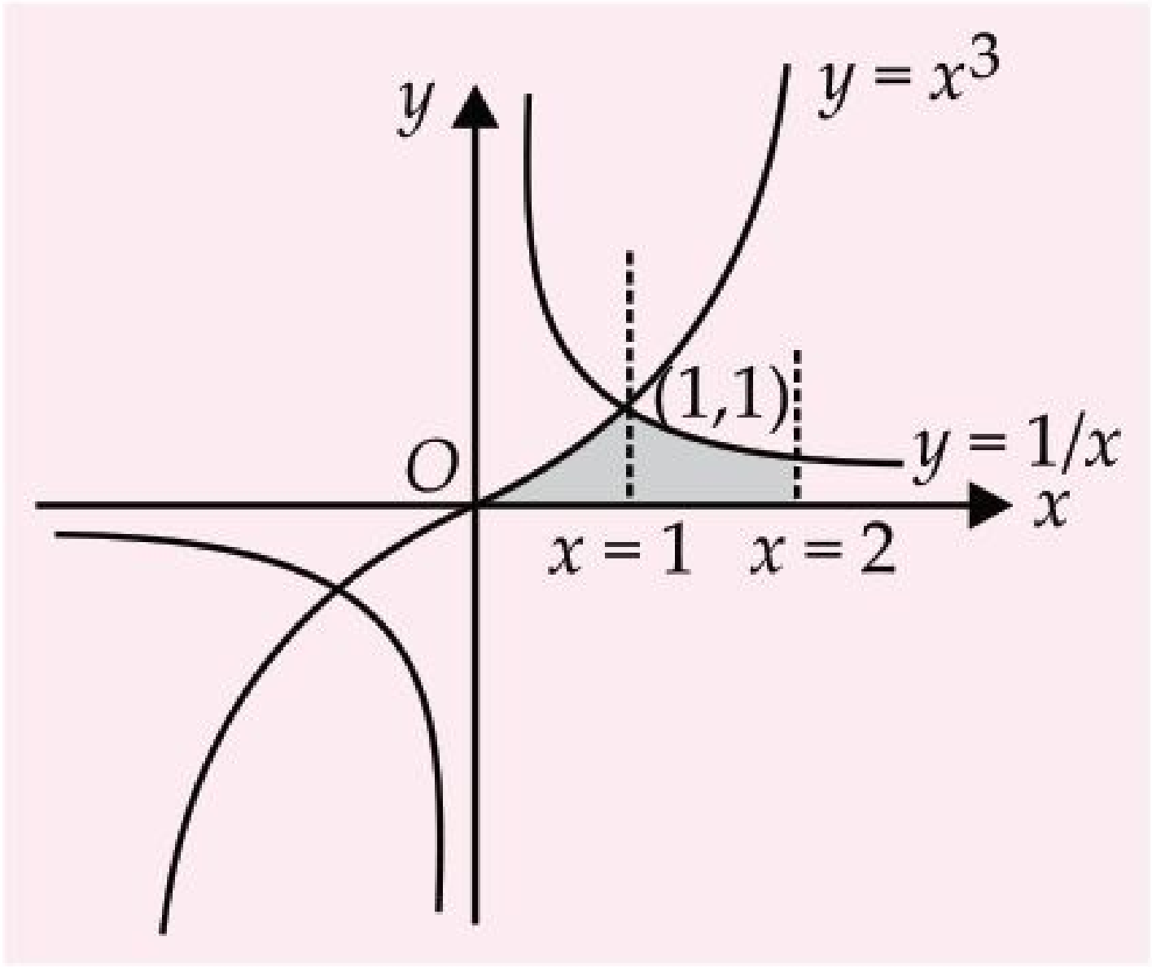
		1				7		2				4		8						
7		8		6	4											6			7	
																5				
					8	4									1			3		4
		5	2										9			7				
	6	4													8			7		
			7	5			3	4	9					1	4		3	8		
8	4					9			2			8	7						9	
6												5								
						1				8		3								
												9		4						
										7										
	9					7	8						1				3		7	
	4			5					4		5			8						
					3	6									6					
			6		1	5		4				9	5				6			
7			8															1		8
						9								4				2		
			9	6		4									1	8		3		
8			1	3				6					7						9	2
3												6				2				

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

$$y=2x^2-2+\left(\frac{4x-\frac{dy}{dx}}{2x}\right)=2x^2-2+2-\frac{1}{2x}\frac{dy}{dx}$$

$$\Rightarrow\ 2yx=2x^2(2x)-\frac{dy}{dx}\ \Rightarrow\ \frac{dy}{dx}=4x^3-2xy$$

10. (b) : Required area $=\int\limits_0^1 x^3dx+\int\limits_1^2 \frac{1}{x}dx$



$$=\left[\frac{x^4}{4}\right]_0^1+\left[\log_e x\right]_1^2=\left(\frac{1}{4}+\log_e 2\right)\text{ sq. units}$$

11. (3) : Given, $\vec{b}=\hat{i}-\hat{j}+3\hat{k},\ \vec{c}=3\hat{i}-\hat{j}+\hat{k}$

$\Rightarrow\ |\vec{b}|=|\vec{c}|=\sqrt{11}$

Let $\vec{a}=x\vec{b}+y\vec{c}$ where x and $y\in R$ and $|\vec{a}|=1$ ...(*)

$\therefore\ \ \vec{a}=x(\hat{i}-\hat{j}+3\hat{k})+y(3\hat{i}-\hat{j}+\hat{k})$

$\vec{a}=\hat{i}(x+3y)+\hat{j}(-x-y)+\hat{k}(3x+y)$

Now $\vec{a}\perp\vec{b}\ \therefore\ \vec{a}\cdot\vec{b}=0$

$\Rightarrow\ [\hat{i}(x+3y)+\hat{j}(-x-y)+\hat{k}(3x+y)]\cdot[\hat{i}-\hat{j}+3\hat{k}]=0$

$\Rightarrow\ x+3y+x+y+3(3x+y)=0$

$\Rightarrow\ 11x+7y=0\ \ \Rightarrow\ \ y=\frac{-11}{7}x$

$$\therefore\ \vec{a}=\hat{i}\left(x+\frac{3(-11x)}{7}\right)+\hat{j}\left(-x+\frac{11}{7}x\right)+\hat{k}\left(3x-\frac{11}{7}x\right)$$

$$=\hat{i}\left(\frac{-26}{7}x\right)+\hat{j}\left(\frac{4x}{7}\right)+\hat{k}\left(\frac{10x}{7}\right)$$

$$\Rightarrow\ |\vec{a}|=1\ \Rightarrow\ \left(\frac{(26)^2}{49}+\frac{4^2}{49}+\frac{10^2}{49}\right)x^2=1$$

$$\Rightarrow\ x^2=\frac{49}{792}\ \therefore\ x=\pm\frac{7}{6\sqrt{22}}$$

$$\therefore\ \vec{a}=\pm\frac{7}{6\sqrt{22}}\left(-\frac{26}{7}\hat{i}+\frac{4}{7}\hat{j}+\frac{10}{7}\hat{k}\right)$$

Now projection of $\vec{a}$ along $\vec{c}=\lambda=\frac{|\vec{a}\cdot\vec{c}|}{|\vec{c}|}$

$$=\left|\frac{\frac{7}{6\sqrt{22}}\left(\frac{-26}{7}\hat{i}+\frac{4}{7}\hat{j}+\frac{10}{7}\hat{k}\right)\cdot(3\hat{i}-\hat{j}+\hat{k})}{\sqrt{11}}\right|=\frac{72}{6\times11\sqrt{2}}$$

$$\therefore\quad \frac{11\sqrt{2}\lambda}{4}=3$$

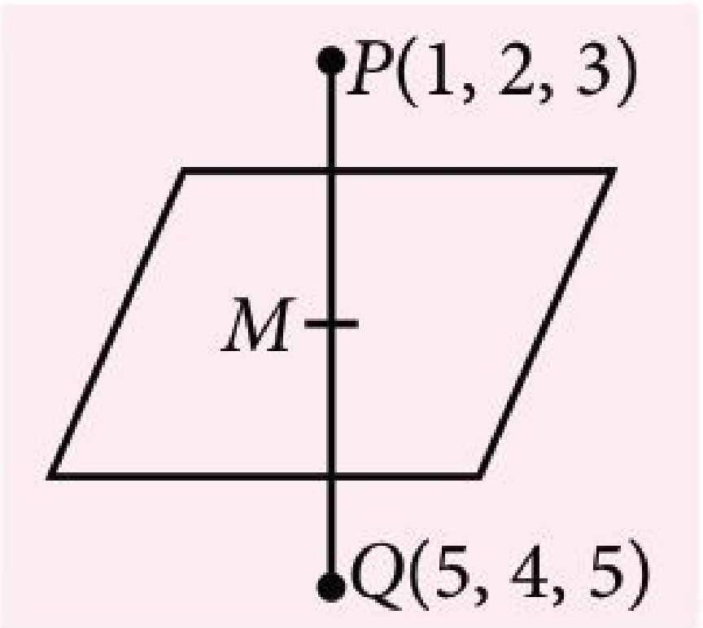
12. (9) : We have, $[\vec{a}+2\vec{b}\ \ \vec{b}+2\vec{c}\ \ \vec{c}+2\vec{a}]$

$=\ (\vec{a}+2\vec{b})\cdot\{(\vec{b}+2\vec{c})\times(\vec{c}+2\vec{a})\}$

$=\ (\vec{a}+2\vec{b})\cdot\{(\vec{b}\times\vec{c})+2(\vec{b}\times\vec{a})+4(\vec{c}\times\vec{a})\}$

$$=[\vec{a}\ \vec{b}\ \vec{c}]+8[\vec{a}\ \vec{b}\ \vec{c}]=9[\vec{a}\,\vec{b}\,\vec{c}]\Rightarrow\ \frac{[\vec{a}+2\vec{b}\ \ \vec{b}+2\vec{c}\ \ \vec{c}+2\vec{a}]}{[\vec{a}\ \vec{b}\ \vec{c}]}=9$$

13. (17) : Here, $M\equiv(3,3,4)$ and normal vector of the plane is parallel to $\overrightarrow{PQ}$.



Hence, $\vec{n}=4\hat{i}+2\hat{j}+2\hat{k}=2(2\hat{i}+\hat{j}+\hat{k})$

$\therefore$ Equation of plane passing through the midpoint M of PQ, is

$2(x-3)+1(y-3)+1(z-4)=0$

$\Rightarrow\ 2x+y+z-13=0$

$\Rightarrow\ a=2,\ b=1,\ c=1,\ d=13$

$\therefore\ \ a+b+c+d=17$

14. (1) : Let B denotes the event of drawing a black ball. Then

$B=(B_1\cup B_2\cup B_3)\cap B$

$=(B_1\cap B)\cup(B_2\cap B)\cup(B_3\cap B)$

$$\therefore\ \ P(B)=P(B_1)P\left(\frac{B}{B_1}\right)+P(B_2)P\left(\frac{B}{B_2}\right)+P(B_3)P\left(\frac{B}{B_3}\right)$$

$$=\frac{1}{6}\times\frac{n}{n+4}+\frac{1}{2}\times\frac{2}{9}+\frac{1}{3}\times\frac{3}{9}=\frac{n}{6(n+4)}+\frac{2}{9}=\frac{3n+4(n+4)}{18(n+4)}$$

$=\frac{23}{90}$ (Given)

$\Rightarrow\ 35n+80=23(n+4)\Rightarrow 12n=12\Rightarrow n=1$

15. (99) : $I=\int_{-1}^{1}\frac{e^{|x|}}{1+a^x}dx$ ...(i)

$I=\int_{-1}^{1}\frac{e^{|0-x|}}{1+a^{0-x}}dx=\int_{-1}^{1}\frac{a^xe^{|x|}}{1+a^x}dx$ ...(ii)

Adding (i) and (ii), we get

$$2I=\int_{-1}^{1}\frac{e^{|x|}(1+a^x)}{(1+a^x)}dx=\int_{-1}^{1}e^{|x|}dx=2\int_0^1e^xdx$$

$\Rightarrow\ I=(e-1)$

$\therefore\ \ \frac{99}{(e-1)}I=99$

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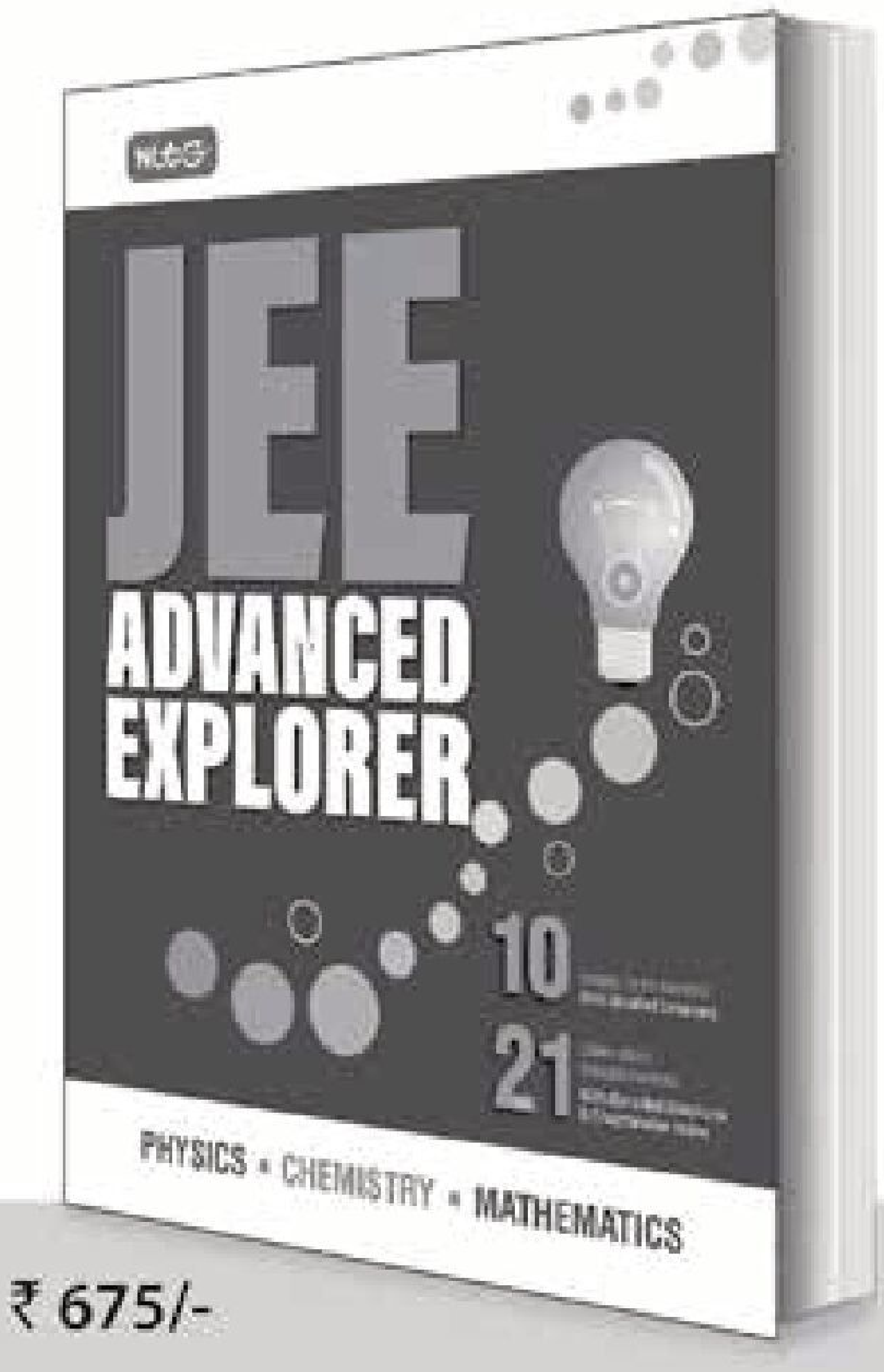
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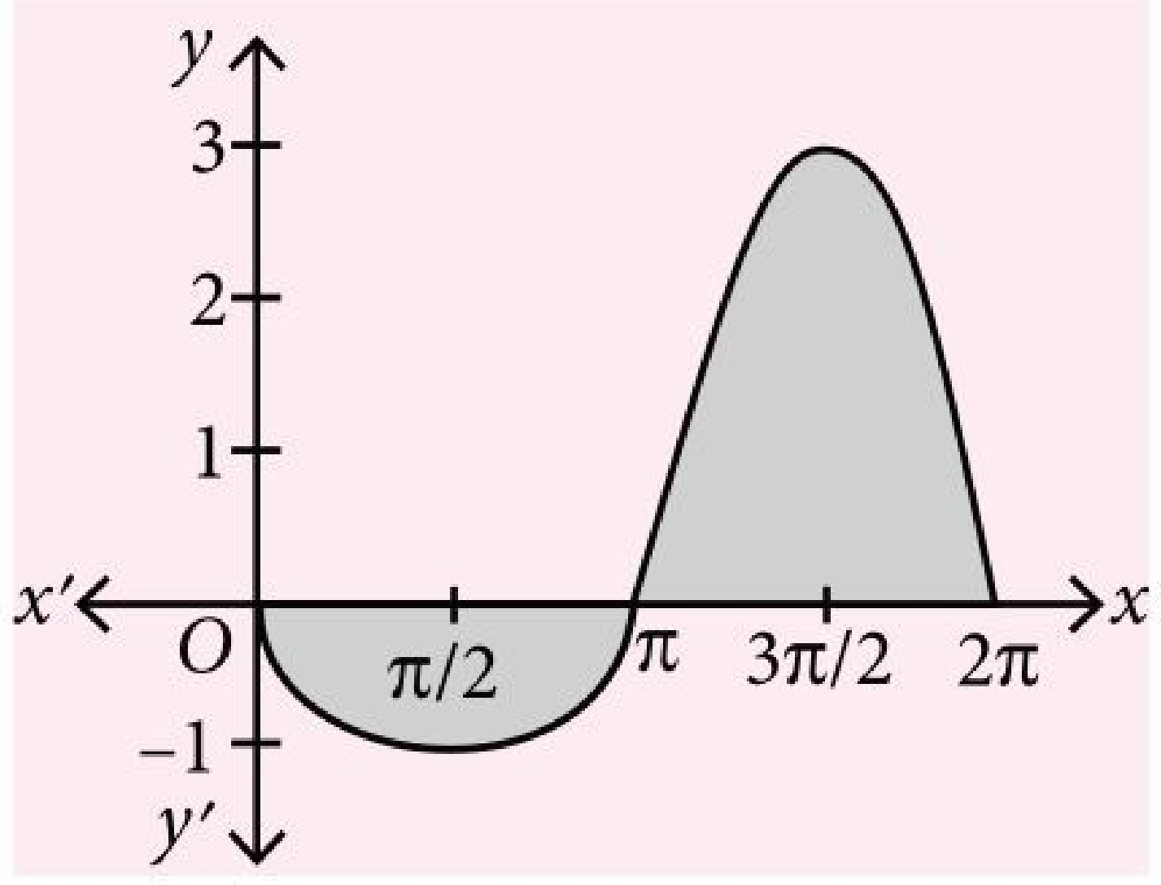
YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Find the area bounded by the curve $y = \sin^2 x - 2\sin x$ and the x-axis, where $x \in [0, 2\pi]$.

(Animesh, Delhi)

Ans. We have, $y = \sin^2 x - 2\sin x = \sin x(\sin x - 2)$



when $y = 0$, $\sin x = 0$

$\Rightarrow \quad x = 0, \pi, 2\pi$

Also, $\sin x - 2 < 0$

$\therefore \quad y > 0$ if $\sin x < 0$

and $y < 0$ if $\sin x > 0$

$\therefore$ Required area

= Area of shaded region

$$= \left|\int_0^{\pi} (\sin^2 x - 2\sin x)\,dx\right| + \int_{\pi}^{2\pi} (\sin^2 x - 2\sin x)\,dx$$

$$= \left|\frac{1}{2}\int_0^{\pi} (1-\cos 2x)\,dx - 2\int_0^{\pi}\sin x\,dx\right| + \int_{\pi}^{2\pi}\left(\frac{1-\cos 2x}{2} - 2\sin x\right)dx$$

$$= \left[\left|\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + 2\cos x\right|\right]_0^{\pi} + \left[\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + 2\cos x\right]_{\pi}^{2\pi}$$

$$= \left|\frac{1}{2}\pi - 4\right| + \frac{2\pi - \pi}{2} + 4 = \left(4 - \frac{\pi}{2}\right) + 4 + \frac{\pi}{2} = 8 \text{ sq. units}$$

2. Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If $P(X = r)/P(X = n - r)$ is independent of n and r, then find the value of p.

(Rajiv Sinha, West Bengal)

Ans. For Binomial distribution,

$$P(X = r) = {}^nC_r\,(p)^r (q)^{n-r}$$

$$= \frac{n!}{(n-r)!\,r!}(p)^r(1-p)^{n-r} \qquad [\because q = 1 - p] \quad \text{...(i)}$$

and $P(X = n - r) = {}^nC_{n-r}\,(p)^{n-r}\,(q)^{n-(n-r)}$

$$= \frac{n!}{(n-r)!\,r!}(p)^{n-r}(1-p)^r \qquad \text{...(ii)}$$

Now, $$\frac{P(X=r)}{P(X=n-r)} = \frac{\dfrac{n!}{(n-r)!\,r!}\,p^r(1-p)^{n-r}}{\dfrac{n!\,p^{n-r}(1-p)^r}{(n-r)!\,r!}}$$

[Using (i) and (ii)]

$$\Rightarrow \quad \frac{P(X=r)}{P(X=n-r)} = \left(\frac{1-p}{p}\right)^{n-r} \times \frac{1}{\left(\dfrac{1-p}{p}\right)^r}$$

Above expression will be independent of n and r, if

$$\frac{1-p}{p} = 1 \Rightarrow p = \frac{1}{2}$$

3. Consider a tetrahedron with faces F_1, F_2, F_3, F_4. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ be the vectors whose magnitudes are respectively equal to areas of F_1, F_2, F_3, F_4 and whose directions are perpendicular to their faces in outward direction. Then find the value of $|\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4|$.

(Ankur Jain, Kerala)

Ans. Let the three dimension edges of the tetrahedron in the right-handed system are $\vec{a}, \vec{b}$ and $\vec{c}$.

$$\therefore \quad \vec{v}_1 = \frac{1}{2}(\vec{a}\times\vec{b}), \vec{v}_2 = \frac{1}{2}(\vec{b}\times\vec{c}), \vec{v}_3 = \frac{1}{2}(\vec{c}\times\vec{a})$$

and $$\vec{v}_4 = \frac{1}{2}[(\vec{c}-\vec{a})\times(\vec{b}-\vec{a})] = \frac{1}{2}[\vec{c}\times\vec{b} - \vec{c}\times\vec{a} - \vec{a}\times\vec{b}]$$

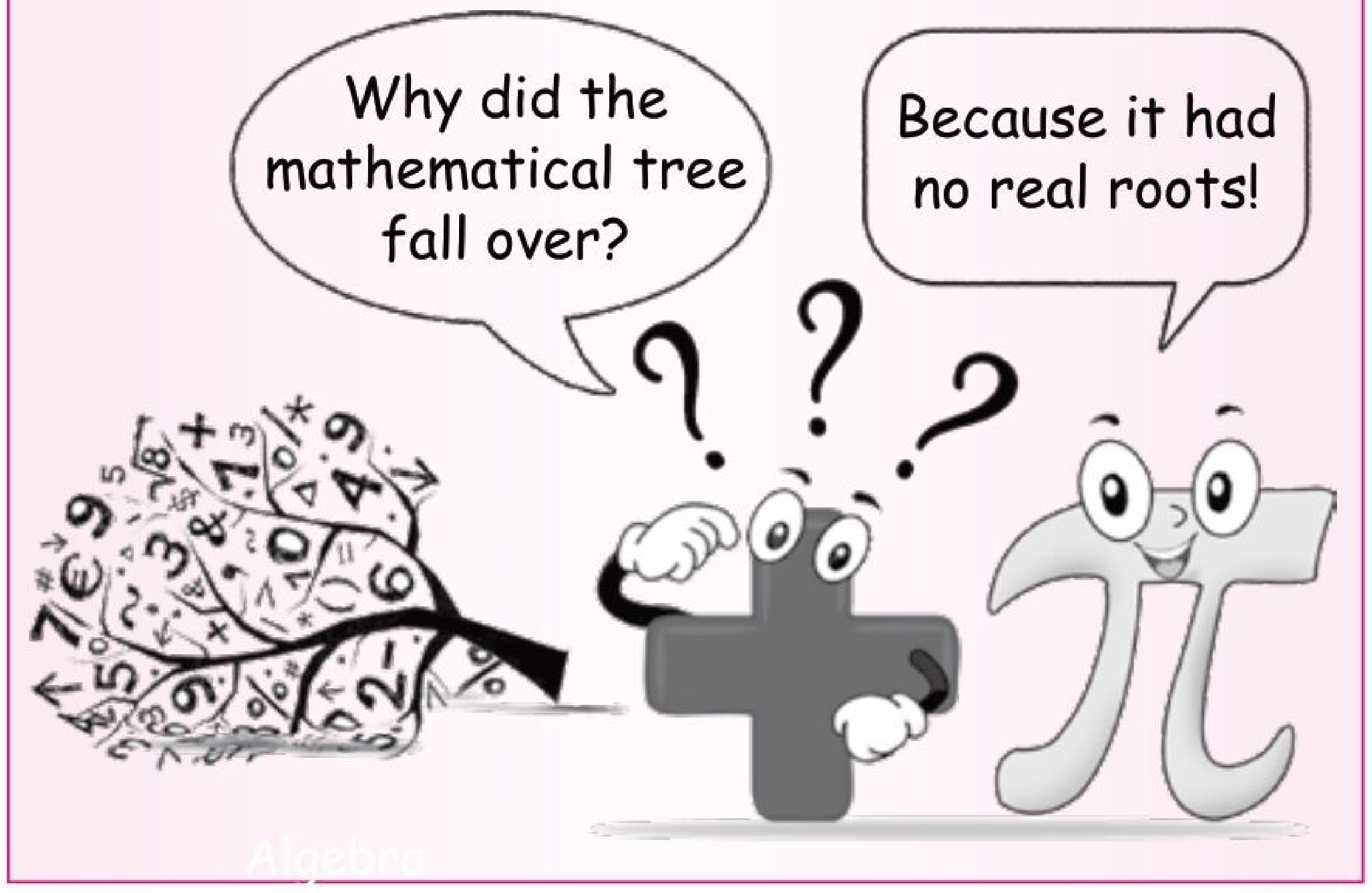
$$\therefore \quad \vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4$$

$$= \frac{1}{2}(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b} - \vec{c}\times\vec{a} - \vec{a}\times\vec{b}) = \vec{0}$$

$$\Rightarrow \quad |\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4| = 0.$$

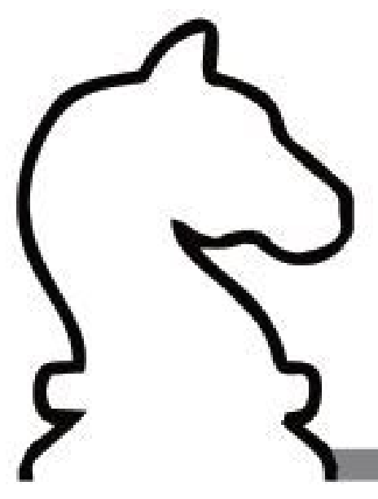
◆ ◆

COMIC CAPSULE



Why did the mathematical tree fall over?
Because it had no real roots!
Algebra

Challenging PROBLEMS ON JEE Main



1. Given that $x_1 + x_2 + x_3 = 0$, $y_1 + y_2 + y_3 = 0$ and $x_1y_1 + x_2y_2 + x_3y_3 = 0$. Then

$$\frac{x_1^2}{x_1^2+x_2^2+x_3^2}+\frac{y_1^2}{y_1^2+y_2^2+y_3^2}=$$

(a) 1/3 (b) 2/3 (c) 1 (d) 4/3

2. If z be a complex number satisfying, $|z|^2 + 2(z + \bar{z}) + 3i(z - \bar{z}) + 4 = 0$ then complex number $z + 3 + 2i$ will lie on

(a) circle with centre $1 - 5i$ and radius 4
(b) circle with centre $1 + 5i$ and radius 4
(c) circle with centre $1 + 5i$ and radius 3
(d) circle with centre $1 - 5i$ and radius 3

3. Let $z = (18 + 26i)$ and $z_0 = a + ib$ is the cube root of z having least positive argument then $a + b =$

(a) 1 (b) 2 (c) 3 (d) 4

4. Consider the two lines L_1, L_2 and a circle C
$L_1 : 2x + 3y + p - 3 = 0$, $L_2 : 2x + 3y + p + 3 = 0$
$C : x^2 + y^2 + 6x + 10y + 30 = 0$, $(p \in I)$
It is given that at least one of the lines L_1, L_2 is a chord of C then the probability that both are chords of C is

(a) 2/7 (b) 3/7 (c) 4/7 (d) 5.7

5. Let $f(x)=\log\left(\frac{x^2+e}{x^2+1}\right)$ and $g(x)=\sqrt{\sin f(x)}+\sqrt{\cos f(x)}$, then range of $g(x)$ is

(a) $(1, 2^{3/4}]$ (b) $(2^{1/2}, 2^{3/4}]$
(c) $(\pi^{1/2}, \pi^{3/4})$ (d) $(e^{1/2}, \pi^{1/2}]$

6. If the parabola $y = ax^2 + bx + c$ has vertex at (4, 2) and $a \in [1, 3]$ then maximum value of product $(a\,b\,c)$ is

(a) 144 (b) 12 (c) −12 (d) −144

7. If one of the roots of the equation

$$\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}=0$$

is $x = 2$, then sum of all other five roots is

(a) $2\sqrt{15}$ (b) −2
(c) $\sqrt{20}+\sqrt{15}-2$ (d) none of these

8. If $f : R \to R$, $g : R \to R$ and $f(x) + f''(x) = -xg(x)f'(x)$ and $g(x) > 0\ \forall\ x \in R$, then $f^2(x) + (f'(x))^2$ has

(a) a minima at $x = 0$
(b) a maxima at $x = 0$
(c) a point of inflexion at $x = 0$
(d) data insufficient

9. All bases of logarithms in which a real positive number can be equal to its logarithm is/ are

(a) $(0, 1) \cup (1, e^{1/e}]$ (b) $(1, e)$
(c) $(1, e^{1/2})$ (d) $[e^{1/e}, e^e)$

10. Let $f(x)=\lim\limits_{m\to 0}\frac{1}{m^4}\cdot\int_0^m\frac{(e^{x+t}-e^x)(\log(1+t))^2}{3+2t^3}dt$

then $f(\log 3) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

11. The non-negative real numbers $x_r (r = 1$ to $5)$ satisfy the following relations :

(1) $\sum_{r=1}^{5} r\cdot x_r = a$ (2) $\sum_{r=1}^{5} r^3 x_r = a^2$ and

(3) $\sum_{r=1}^{5} r^5 x_r = a^3$, then number of possible values of a is/are

(a) 0 (b) 1 (c) 6 (d) 12

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

12. $\int \frac{dx}{\sqrt{\cos^3 x \cos(x-\alpha)}}$ is equal to

(a) $2\sqrt{\tan x \sin\alpha + \cot\alpha} + c$

(b) $2\,\text{cosec}\,\alpha\sqrt{\tan x \sin\alpha + \cos\alpha} + c$

(c) $2\sqrt{\sin\alpha + \tan x \cos\alpha} + c$

(d) $2\,\text{cosec}\,\alpha\sqrt{\tan x \cos\alpha + \sin\alpha} + c$

13. The natural number n for which $2^8 + 2^{11} + 2^n$ is a perfect square is

(a) 11 (b) 12 (c) 14 (d) 15

14. If $f(x)$ is a differentiable function defined $\forall\, x \in R$ such that $(f(x))^3 = x - f(x)$ then $\int_0^{\sqrt{2}} f^{-1}(x)dx =$

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

15. If O is the origin and A and B are respectively (a_1, b_1) and (a_2, b_2), then $OA \cdot OB \sin(AOB)$ is equal to

(a) $a_1a_2 + b_1b_2$ (b) $a_1b_2 + a_2b_1$

(c) $b_1a_2 - b_2a_1$ (d) $a_1a_2 - b_1b_2$

16. Suppose a and b are single digit positive integers chosen independently and at random. The probability that the point (a, b) lies above the parabola $y = ax^2 - bx$ is

(a) $\frac{17}{81}$ (b) $\frac{19}{81}$ (c) $\frac{21}{81}$ (d) $\frac{23}{81}$

17. Let $f(x) = ax^2 + bx + c$, $a, b, c \in I$. Let $f(1) = 0$, $f(7) \in (50, 60)$, $f(8) \in (70, 80)$ then $f(2) \in$

(a) $(-2, 0)$ (b) $(0, 10)$ (c) $(1, 12)$ (d) $(20, 30)$

18. In ΔABC, H is the orthocentre and $AH \cdot BH \cdot CH = 3$ and $AH^2 + BH^2 + CH^2 = 7$, then sum of the possible circumradius (R) of the ΔABC is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

SOLUTIONS

1. (b) : Consider three vectors

$\vec{n}_1 = (x_1, x_2, x_3), \vec{n}_2 = (y_1, y_2, y_3)$ and $\vec{n}_3 = (1, 1, 1)$.

From the given data,

$$\vec{n}_1 \cdot \vec{n}_2 = 0, \vec{n}_2 \cdot \vec{n}_3 = 0 \text{ and } \vec{n}_1 \cdot \vec{n}_3 = 0$$

i.e., $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are mutually $\perp^r$ vectors.

Now, $\frac{x_1^2}{x_1^2 + x_2^2 + x_3^2}, \frac{y_1^2}{y_1^2 + y_2^2 + y_3^2}$ and $\frac{1}{3}$ are the squares of the projections of the vector (1, 0, 0) onto the direction of $\vec{n}_1, \vec{n}_2, \vec{n}_3$ respectively and hence their sum = 1

$$i.e., \frac{x_1^2}{x_1^2 + x_2^2 + x_3^2} + \frac{y_1^2}{y_1^2 + y_2^2 + y_3^2} + \frac{1}{3} = 1$$

2. (c) : Given equation of circle is $|z|^2 + z(2 + 3i) + \bar{z}(2 - 3i) + 4 = 0$, with centre at $a = -(2 - 3i)$,

radius = $\sqrt{a\bar{a} - b} = 3$, where $b = 4$

Let $\omega = z + 3 + 2i = z + 2 - 3i + 1 + 5i$,

$|\omega - 1 - 5i| = |z + 2 - 3i| = 3$

3. (d) : $z = 18 + 26i = 10\sqrt{10}\,[\cos\theta + i\sin\theta]$

where $\tan\theta = \frac{13}{9} \Rightarrow \tan\left(\frac{\theta}{3}\right) = \frac{1}{3}$

and $z_0 = z^{1/3} = \sqrt{10}\,[\cos(\theta/3) + i\sin(\theta/3)]$

$$z_0 = \sqrt{10}\left[\frac{3}{\sqrt{10}} + \frac{i}{\sqrt{10}}\right] = 3 + i$$

$\Rightarrow a + b = 4$

4. (b) : For L_1 to be a chord of the circle, possible integral value of p are {17, 18, ..., 31}. Similarly, for L_2 to be a chord, possible integral value of p are {11, 12, ..., 25}.

So, in total possible $p = 21$ and common values are 9.

Hence, probability = $\frac{9}{21} = \frac{3}{7}$

5. (a) : Notice that $\frac{x^2 + e}{x^2 + 1} \in [1, e]$

Hence, $f(x) \in (0, 1]$

So, $g(\alpha) = \sqrt{\sin\alpha} + \sqrt{\cos\alpha},\ \alpha \in (0, 1]$

$g'(\alpha) = 0$ gives $\alpha = \pi/4$

So, $g(x) \in (1, 2^{3/4}]$

6. (d) : From given data, $\frac{-b}{2a} = 4$ and $\frac{-D}{4a} = 2$

So, $c = 2 + 16a$

and $E = abc = -16(a^2 + 8a^3)$

So, $\frac{dE}{da} = -16(2a + 24a^2) < 0$, for all $a \in [1, 3]$

$\Rightarrow E(a)$ is decreasing on [1, 3].

Hence, $E_{\max.} = -16(1^2 + 8 \cdot 1^3) = -144$

7. (b) : Let $x^2 - 13 = t$, then $t^3 - 67t + 126 = 0$

$\Rightarrow t = -9, 7, 2 \Rightarrow x = \pm 2, \pm\sqrt{20}, \pm\sqrt{15}$.

8. (b) : Let $h(x) = (f(x))^2 + (f'(x))^2$

So, $h'(x) = -2x\, g(x) \cdot (f'(x))^2$

$\therefore\ h'(0^+) > 0$ and $h'(0^-) < 0$

9. (a) : We require $a \in R - \{1\}$ such that $x = \log_a x$

i.e. $f(x) = \frac{\log x}{x} = \log a$

$f'(x) = 0 \Rightarrow x = e$

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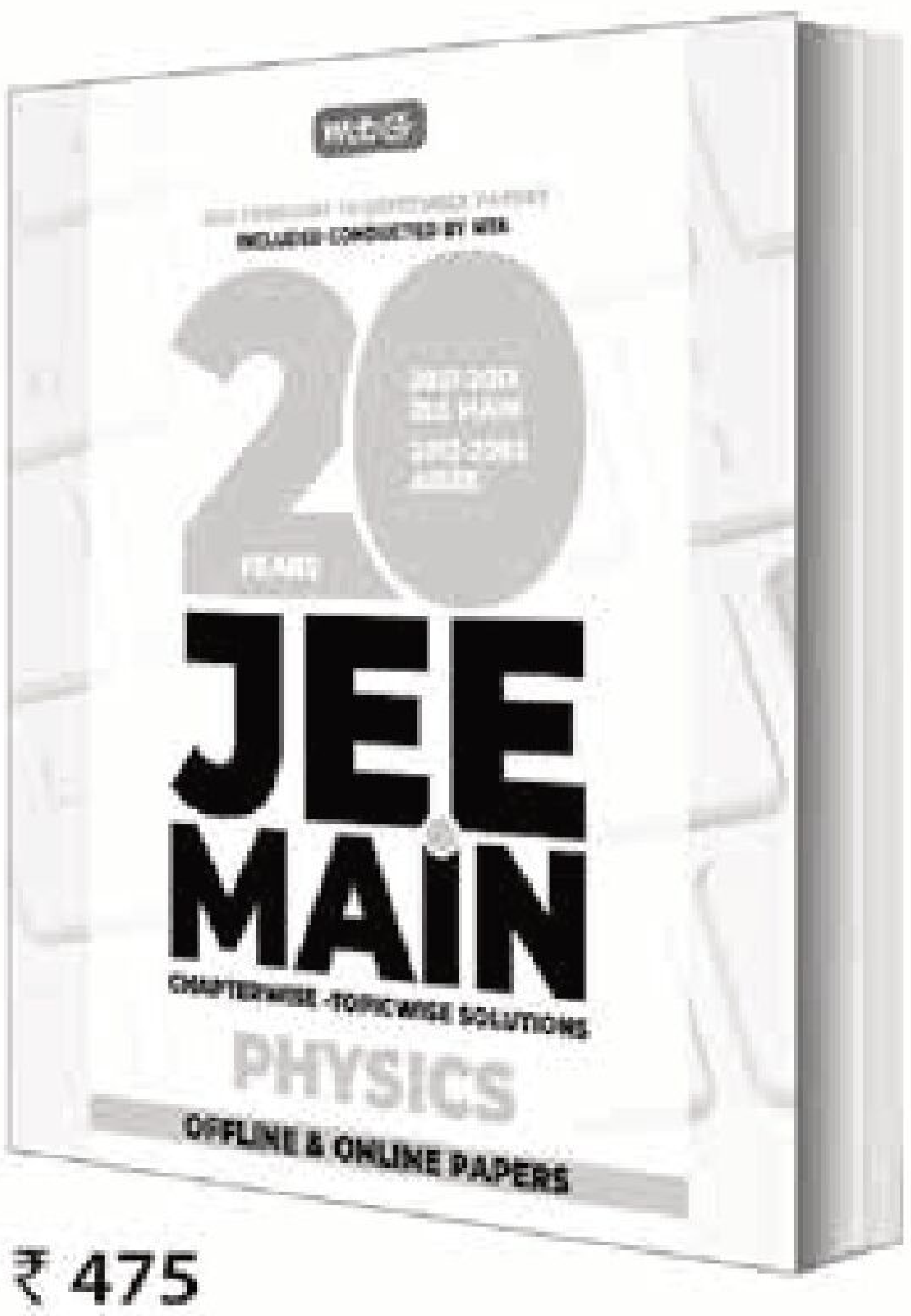
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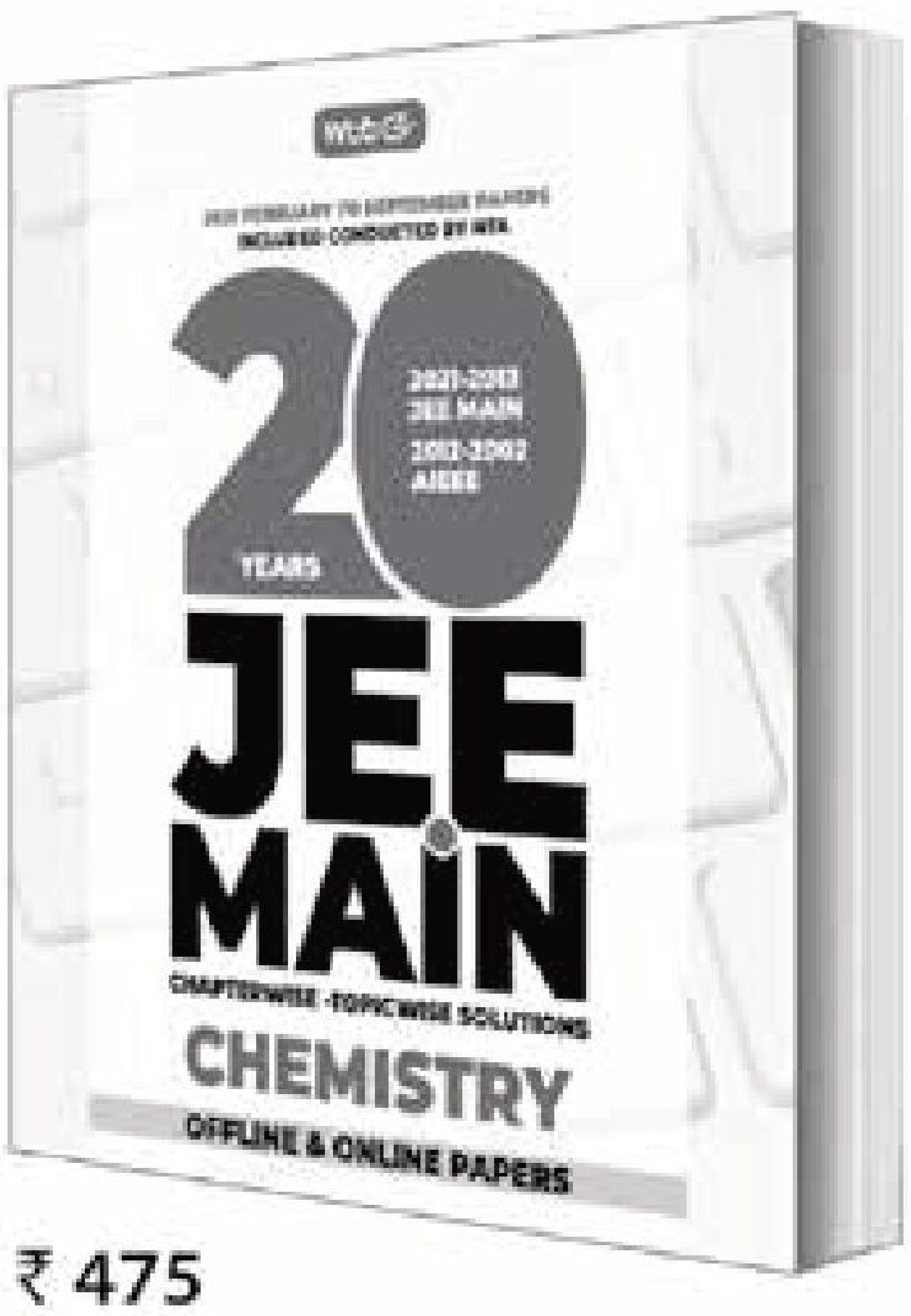
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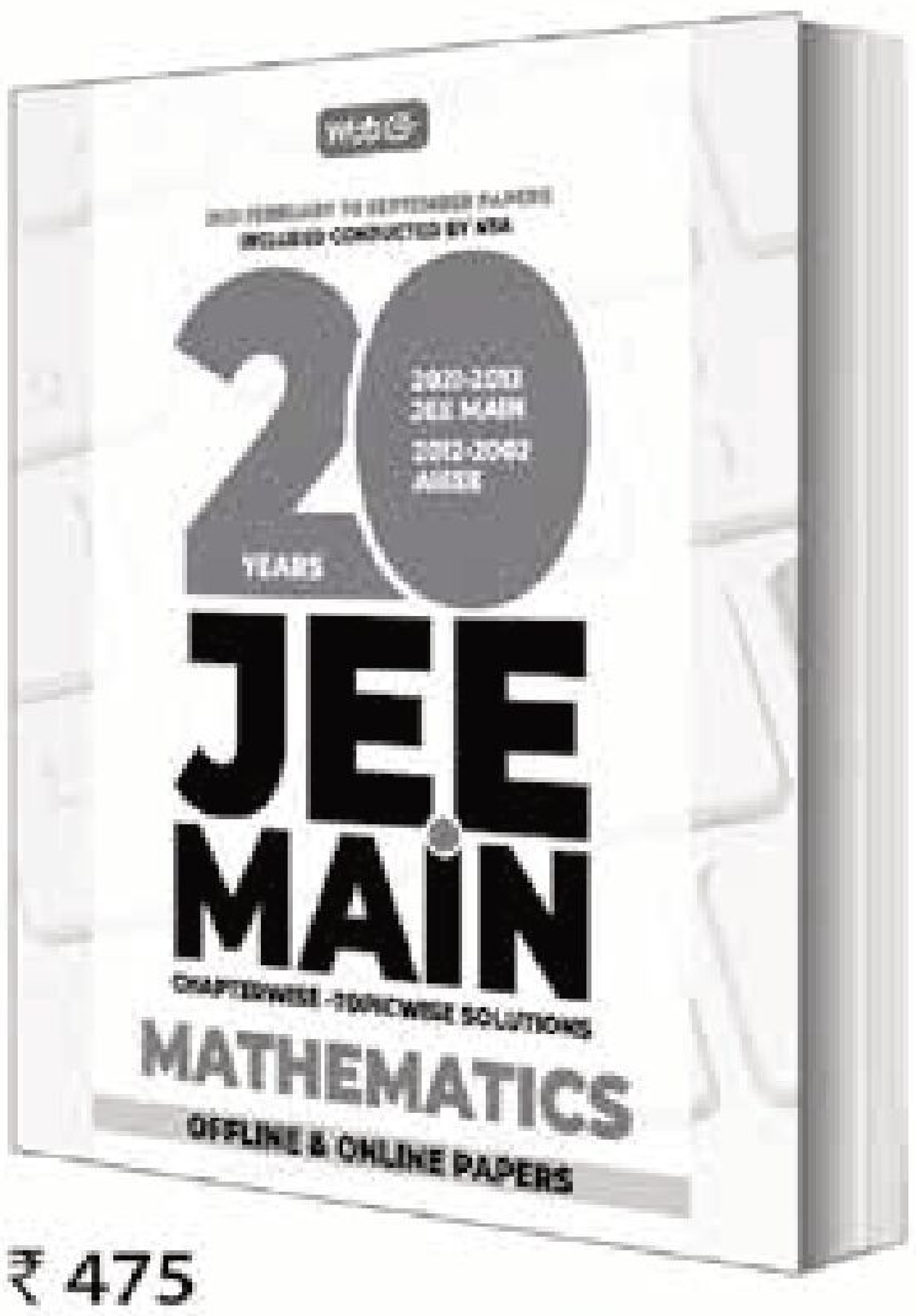
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HIGHLIGHTS:

- Chapterwise-Topicwise Questions (Online & Offline) of last 20 years' (2021-2002) of JEE Main / AIEEE
- Chapterwise-Topicwise segregation of questions to help you assess the level of effort required to succeed
- An unmatched series with more than 1552 pages filled with detailed solutions by experts

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So, max. $f(x) = f(e) = 1/e$. So, $a_{\max.} = e^{1/e}$

Hence, $a \in (0, 1) \cup (1, e^{1/e}]$

10. (c) : $$f(x)=e^x\cdot\lim_{m\to0}\frac{\displaystyle\int_0^m\frac{(e^t-1)\cdot(\log(1+t))^2}{2t^3+3}dt}{m^4}\quad\left(\frac{0}{0}\right)$$

$$\Rightarrow\quad f(x)=e^x\cdot\lim_{m\to0}\frac{(e^m-1)\cdot(\log(1+m))^2}{(3+2m^3)\cdot4m^3}$$

$$\Rightarrow\quad f(x)=e^x\cdot\lim_{m\to0}\frac{e^m-1}{m}\cdot\left(\frac{\log(1+m)}{m}\right)^2\cdot\frac{1}{4}\cdot\frac{1}{3+2m^3}$$

$$\Rightarrow\quad f(x)=\frac{e^x}{12},\text{ so, } f(\log 3)=\frac{3}{12}=\frac{1}{4}$$

11. (c) : Notice that

$$\sum_{r=1}^{5}r(a-r^2)^2x_r=a^2\sum_{r=1}^{5}r\cdot x_r-2a\sum_{r=1}^{5}r^3x_r+\sum_{r=1}^{5}r^5x_r$$

$= a^2(a) - 2a(a^2) + (a^3) = 0$

Since, L.H.S. terms are non-negative.

Hence, each term in L.H.S. is zero.

So, possible values of a are $\{0, 1, 4, 9, 16, 25\}$.

12. (b) : Let $I = \displaystyle\int\frac{dx}{\sqrt{\cos^3 x(\cos x\cos\alpha+\sin x\sin\alpha)}}$

$$=\int\frac{\sec^2 x\,dx}{\sqrt{\cos\alpha+\tan x\sin\alpha}},\text{ Put }\tan x\sin\alpha+\cos\alpha=t^2$$

$$\Rightarrow\sec^2 x\,dx=\frac{2t\,dt}{\sin\alpha}\quad\therefore\ I=\int\frac{2t\,dt}{\sin\alpha.t}=2\operatorname{cosec}\alpha.t$$

$$=2\operatorname{cosec}\alpha\sqrt{\tan x\sin\alpha+\cos\alpha}+c$$

13. (b) : $2^8 + 2^{11} + 2^n = 2^8(9 + 2^{n-8})$

Hence, $9 + 2^{n-8}$ should be a perfect square.

So, $9 + 2^{n-8} = k^2$ (say) *i.e.* $2^{n-8} = (k + 3)(k - 3)$

So, $(k + 3)$ and $(k - 3)$ are both powers of 2.

$\Rightarrow$ $k = 5$ being the only possibility. Hence, $n = 12$

14. (b) : $(f(x))^3 + f(x) = x$. Hence, $f^{-1}(x) = x^3 + x$

$$\int_0^{\sqrt{2}}f^{-1}(x)dx=\left[\frac{4}{4}+\frac{2}{2}\right]=2$$

15. (c) : $\sin(\theta_1 - \theta_2) = \sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2$

$$=\frac{b_1}{\sqrt{a_1^2+b_1^2}}\cdot\frac{a_2}{\sqrt{a_2^2+b_2^2}}-\frac{a_1}{\sqrt{a_1^2+b_1^2}}\cdot\frac{b_2}{\sqrt{a_2^2+b_2^2}}$$

$$\Rightarrow\quad\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}\sin(\theta_1-\theta_2)=b_1a_2-b_2a_1$$

$\Rightarrow$ $OA\cdot OB\sin(AOB) = b_1a_2 - b_2a_1$.

16. (b) : If (a, b) lies above the curve then $b > y(a)$

$$i.e., b>a^3-ba\quad\Rightarrow\quad b>\frac{a^3}{a+1}$$

The only possibilities are $a = 1, 2, 3$

For $a = 1$, $b = 1, 2, 3, ...9$; For $a = 2$, $b = 3, 4, ... 9$

For $a = 3$, $b = 7, 8, 9$. So, in total there are 19 points out of $9 \times 9 = 81$ points.

Hence, required probability = 19/81

17. (a) : $f(1) = 0 \Rightarrow a + b + c = 0$

$f(7) \in (50, 60) \Rightarrow 50 < 49a + 7b + c < 60$

or $50 < 48a + 6b < 60$ *i.e.,* $8a+b\in\left(\frac{25}{3},10\right)$

i.e., $8a + b = 9$

Similarly, $9a + b = 11$

Hence, $a = 2$, $b = -7$, $c = 5$

i.e., $f(x) = 2x^2 - 7x + 5 \quad\Rightarrow\quad f(2) = 8 - 14 + 5 = -1$

18. (b) : If ΔABC is acute, then Cosine rule gives,

$AB^2 = AH^2 + BH^2 - 2AH\cdot BH\cos(\pi - C)$ ...(i)

and $AB = 2R\sin C$, $CH = 2R\cos C$

$\Rightarrow$ $AB^2 + CH^2 = 4R^2$ ...(ii)

From (i) and (ii),

$$4R^2=AH^2+BH^2+CH^2+\frac{AH\cdot BH\cdot CH}{R}$$

Now, A.T.Q., $4R^2=7+\frac{3}{R}$ *i.e.,* $4R^3-7R-3=0$

i.e., $(R + 1)(2R + 1)(2R - 3) = 0$

Since, $3 = AH\cdot BH\cdot CH < (2R)^3 \Rightarrow R = 1$

Similarly when ΔABC is obtuse, we have $R = 3/2$

So, sum of possibilities of $R=\frac{3}{2}+1=\frac{5}{2}$

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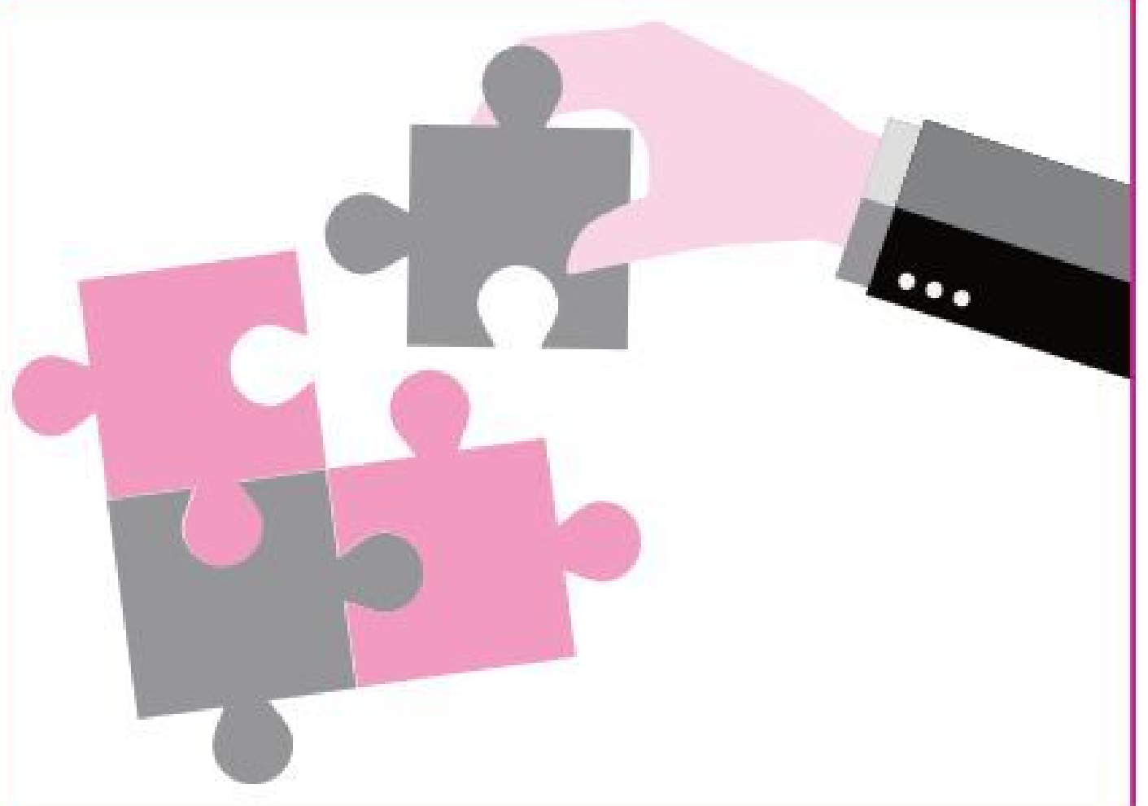
PUZZLE CORNER

SOLUTION · JANUARY 2022



17+ 6	5	15+ 4	3	2÷ 2	1
12× 2	6	3	5	16× 1	4
3	2	1− 1	2− 6	4	1− 5
12× 1	3	2	4	90× 5	6
4	20× 1	14+ 5	2	6	3
5	4	6	1	1− 3	2

OLYMPIAD CORNER

1. Prove that for all positive intergers n, $p_n(x) = x^n - 2$ is irreducible in $Q[x]$.
2. If the sum of the proper divisors of n (that is, the divisors of n that are less than n)is n, then n is called a perfect number and the sum of the divisors of n, $\sigma(n)$ is $2n$, then find all even perfect numbers.
3. Let r be a line in the plane and let ABC be a triangle contained in one of the half-planes determined by r. Let A', B', C', be the points symmetric to A, B, C with respect to r; draw the line through A' parallel to BC, the line through B' parallel to AC and the line through C' parallel to AB. Show that these three lines have a common point.
4. Determine all the possible values of the sum of the digits of the perfect squares.
5. Little sam is a unique child and his math marks show it. On four tests this year his scores out of 100 were all two-digit numbers made up of eight different non-zero digits. Also, the average of these scores is the same as the average if each score is reversed and this average is an integer none of whose digits is equal to any of the digits in the scores. What is sam's average?
6. Prove the inequality

$$\left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right) \ge \left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{a_i b_i}{a_i+b_i}\right)$$

   for any positive numbers $a_1, ..., a_n, b_1, ..., b_n$.
7. A circular billiard table is given with a cue ball at the circumference. It is shot at an angle of α to the line from the ball to the centre of the table. For what angles α will the ball come back to this point, assuming the ball keeps going indefinitely?

SOLUTIONS

1. This follows directly from Eisenstein's criterion, but we will take a more basic approach. By Demoivre's theorem, the n roots of $p_n(x)$ (in C) are $2^{\frac{1}{n}}\operatorname{cis}\left(\frac{2k\pi}{n}\right)$, $k = 0, 1, 2, ..., n - 1$, so that

$$p_n(x) = \left(x - 2^{\frac{1}{n}}\operatorname{cis} 0\right)\left(x - 2^{\frac{1}{n}}\operatorname{cis}\left(\frac{2\pi}{n}\right)\right)....$$

$$\left(x - 2^{\frac{1}{n}}\operatorname{cis}\left(\frac{2(n-1)\pi}{n}\right)\right)$$

Suppose $p_n(x) = f(x)\,g(x)$ for some $f(x), g(x) \in Q[x]$, with $\deg f(x) = m$ and $n > m > 0$. (Note : We only need to consider $n \ge 2$). Let the roots of $f(x)$, respectively $g(x)$, be $w_1, w_2, ..., w_m$, respectively $w_{m+1}, w_{m+2}, ..., w_n$, so $w_1, w_2, ..., w_n$ is permutation of the roots of $p_n(x)$.

Let $w_i = 2^{1/n}\operatorname{cis}\theta_i$, and let $\theta = \sum_{k=1}^{m}\theta_i$.

Since, $(-1)^m\prod_{k=1}^{m} w_k$ is constant term of $f(x)$, so

$$(-1)^m\prod_{k=1}^{m} w_k = (-1)^m 2^{\frac{m}{n}}\prod_{k=1}^{m}\operatorname{cis}\theta_k = (-1)^m 2^{\frac{m}{n}}\operatorname{cis}\theta \in Q.$$

But $\operatorname{cis}\theta = \cos\theta + i\sin\theta \in Q$ implies that $\sin\theta = 0$ and further, that $\cos\theta = \pm 1$. Therefore, the constant term of $f(x)$ is $\pm 2^{\frac{m}{n}}$. It is easy to show that $2^{\frac{m}{n}} \notin Q$, a contradiction. Hence, $x^n - 2$ is irreducible in $Q[x]$.

2. By the Fundamental Theorem of Arithmetic, we can express n uniquely, up to permutations, is the form $p_1^{a_1}p_2^{a_2}\cdots p_k^{a_k}$, where the p_i are distinct primes and the a_i are positive integers. Then, all divisors of n are of the form $p_1^{a_1}p_2^{a_2}...p_k^{a_k}$, where $0 \le a_i \le a_i$.

$$\text{Hence, } \sigma(n) = \prod_{j=1}^{k}\left(1 + p_j + p_j^2 + ... + p_j^{a_j}\right)$$

$$= \left(\frac{p_1^{a_1+1}-1}{p_1-1}\right)\left(\frac{p_2^{a_2+1}-1}{p_2-1}\right)...\left(\frac{p_k^{a_k+1}-1}{p_k-1}\right).$$

It follows that if gcd $(m, n) = 1$, then, $\sigma(mn) = \sigma(m)\sigma(n)$.
We claim that if 2^n-1 is prime, then $2^{n-1}(2^n - 1)$ is a perfect number.
In such a case, we have

$$\sigma(2^{n-1}(2^n-1)) = \frac{2^n-1}{2-1}\cdot\frac{(2^n-1)^2-1}{(2^n-1)-1}$$

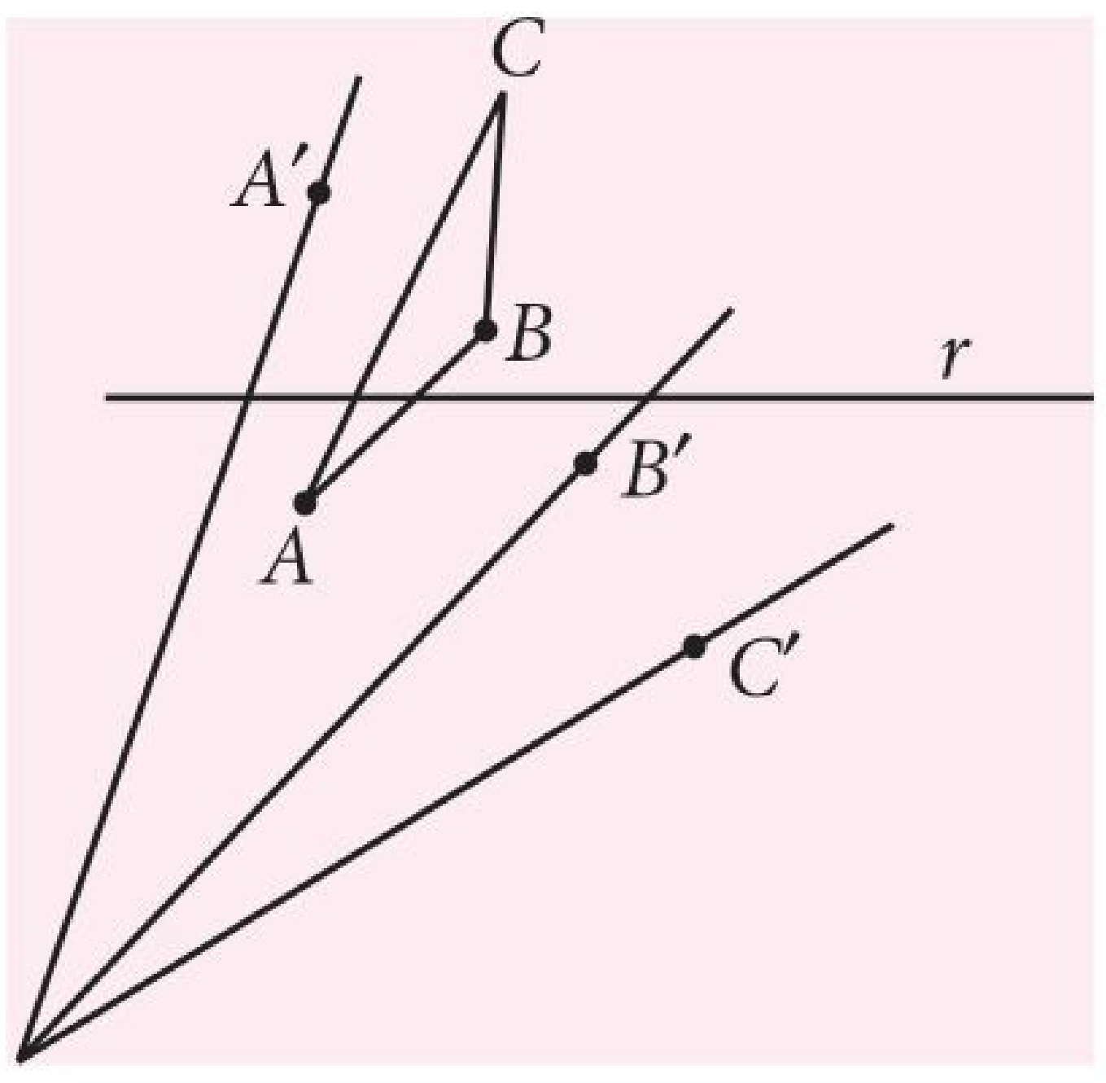
$$= (2^n-1)(2^n-1+1) = 2\cdot 2^{n-1}(2^n-1).$$

Now consider an arbitrary even perfect number n.
Then, $n = 2^k m$, where $k \geq 1$ and gcd $(2^k, m) = 1$.
Also, $2^{k+1}m = \sigma(2^k m) = \sigma(2^k)\,\sigma(m) = (2^{k+1} - 1)\,\sigma(m)$.
Now, $2^{k+1} - 1$ is odd, so $(2^{k+1} - 1) \mid m$.
Let $m = d(2^{k+1} - 1)$, so d is divisor of m.

Then, $\sigma(m) = \dfrac{2^{k+1}m}{2^{k+1}-1} = m + \dfrac{m}{2^{k+1}-1} = m + d.$

But $\sigma(m)$ is defined as the sum of all of the divisors of m. Therefore, m only has the two divisors m and d, so m is prime, d is 1, and so $2^{k+1} - 1$ is prime.
Hence, we have shown that all even perfect numbers are of the form $2^{n-1}(2^n - 1)$, where $2^n - 1$ is prime.

3. Here, there is no need that ΔABC be contained in one of the half planes determined by r.



If the coordinates of A are (x_1, y_1), the coordinates of B are (x_2, y_2) and those of C are (x_3, y_3) in a cartesian coordinate system with r as x-axis then A' has coordinates $(x_1, -y_1)$, B' has coordinates $(x_2, -y_2)$ and C' has coordinates $(x_3, -y_3)$.
The equation of the line through A' parallel to BC is
$(y_2 - y_3)x - (x_2 - x_3)y - x_1(y_2 - y_3) - y_1(x_2 - x_3) = 0$
The equation of the line through B' parallel to CA is
$(y_3 - y_1)x - (x_3 - x_1)y - x_2(y_3 - y_1) - y_2(x_2 - x_1) = 0$
The equation of the line through C' parallel to AB is
$(y_1 - y_2)x - (x_1 - x_2)y - x_3(y_1 - y_2) - y_3(x_1 - x_2) = 0$
Since the three equations when added together vanish identically, the lines represented by them meet in a point. Its coordinates are found, by solving between any two, to be

$$\left(\frac{(x_1^2+x_2x_3)(y_2-y_3)+(x_2^2+x_3x_1)(y_3-y_1)+(x_3^2+x_1x_2)(y_1-y_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)},\right.$$
$$\left.\frac{(y_1^2+y_2y_3)(x_2-x_3)+(y_2^2+y_3y_1)(x_3-x_1)+(y_3^2+y_1y_2)(x_1-x_2)}{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)}\right)$$

4. The squares can only be 0, 1, 4 or 7 mod 9.
Thus the sum of the digits of a perfect square cannot be 2, 3, 5, 6 or 8 mod 9, since the number itself would then be 2, 3, 5, 6 or 8 mod 9.
We shall show that the sum of the digits of a perfect square can take every value of the form 0, 1, 4 or 7 mod 9.

$$(10^m-1)^2 = 10^{2m} - 2\cdot 10^m + 1 = 9\,\underbrace{9\ldots9}_{m-1}\,8\,\underbrace{0\ldots0}_{m-1}\,1, m \geq 1.$$

The sum of the digits is $9m$, giving all the values greater than or equal to 9 congruent to 0 mod 9

$$(10^m-2)^2 = 10^{2m} - 4\cdot 10^m + 4 = 9\,\underbrace{9\ldots9}_{m-1}\,6\,\underbrace{0\ldots0}_{m-1}\,4, m \geq 1.$$

The sum of the digits is $9m + 1$, which gives all values greater than or equal to 10 congruent to 1 mod 9

$$(10^m-3)^2 = 10^{2m} - 6\cdot 10^m + 9 = 9\,\underbrace{9\ldots9}_{m-1}\,4\,\underbrace{0\ldots0}_{m-1}\,9, \; m \geq 1.$$

The sum of the digits is $9m + 4$, which takes every value greater than or equal to 13 which is congruent to 4 mod 9

$$(10^m-5)^2 = 10^{2m} - 10^{m+1} + 25 = \underbrace{9\ldots9}_{m-1}\,0\,\underbrace{0\ldots0}_{m-1}\,2\,5.$$

The sum of the digits is $9(m - 1) + 7 = 9m - 2$, from which we get every value greater than or equal to 7 congruent to 7 mod 9.
Note : We have taken care of all the integers apart from 0, 1, 4, which are the sums of the digits of 0^2, 1^2 and 2^2 respectively.

5. Let the four marks be $10a + b$, $10c + d$, $10e + f$ and $10g + h$.
Then, $10a + b + 10c + d + 10e + f + 10g + h$
$= 10b + a + 10d + c + 10f + e + 10h + g$,
So, $9(a + c + e + g) = 9(b + d + f + h)$
and $a + c + e + g = b + d + f + h$. ...(i)
Therefore, the average of the four marks is

$$\frac{10a+b+10b+a+10c+d+10d+c+10e+f+10f+e+10g+h+10h+g}{4\cdot 2}$$

$$= \frac{11(a+b+c+d+e+f+g+h)}{8} \quad \text{...(ii)}$$

The average must consist only of k's and 0's, the digits not equal to a, b, c, d, e, f, g or h.

Thus, from (ii) $\dfrac{11(1+2+\ldots+9-k)}{8} = \dfrac{11(45-k)}{8}$,

and so $8 \mid (45 - k)$. Since $k \leq 9$, the only solution for k is $k = 5$. Therefore, Sam's average is $11(40)/8 = 55$.

6. We know from Cauchy's inequality that

$$\left(\sum_{i=1}^{n}(a_i-b_i)\right)^2 \le \left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i-b_i)^2}{a_i+b_i}\right)$$

Therefore

$$\left(\sum_{i=1}^{n}a_i\right)\left(\sum_{i=1}^{n}b_i\right)=\frac{1}{4}\left(\left(\sum_{i=1}^{n}a_i+\sum_{i=1}^{n}b_i\right)^2-\left(\sum_{i=1}^{n}a_i-\sum_{i=1}^{n}b_i\right)^2\right)$$

$$=\frac{1}{4}\left(\left(\sum_{i=1}^{n}(a_i+b_i)\right)^2-\left(\sum_{i=1}^{n}(a_i-b_i)\right)^2\right)$$

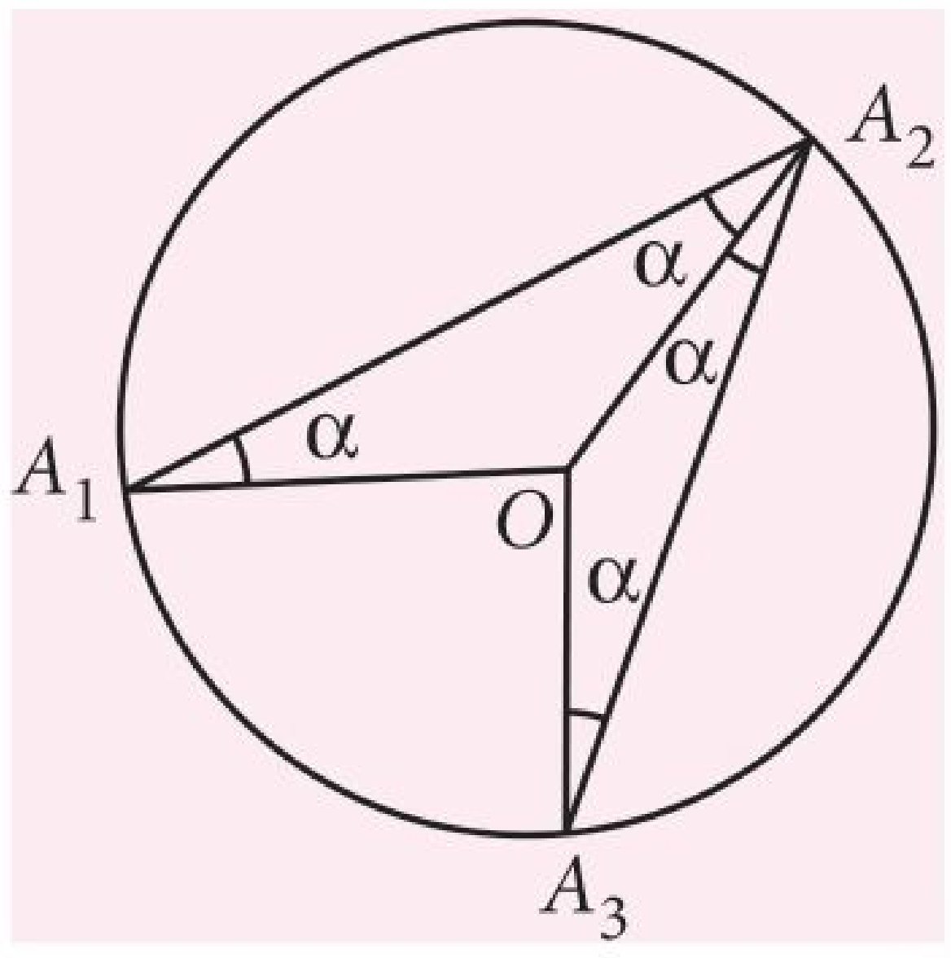
$$\ge\frac{1}{4}\left(\left(\sum_{i=1}^{n}(a_i+b_i)\right)^2-\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i-b_i)^2}{a_i+b_i}\right)\right)$$

$$=\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{(a_i+b_i)^2-(a_i-b_i)^2}{4(a_i+b_i)}\right)$$

$$=\left(\sum_{i=1}^{n}(a_i+b_i)\right)\left(\sum_{i=1}^{n}\frac{a_ib_i}{a_i+b_i}\right)$$

which completes the proof.

7. We let the ball start at A_1 as shown. Then $\angle O A_1A_2 = \angle A_1A_2 O = \alpha$ (equal radii) and $\angle A_1O A_2 = 180° - 2\alpha$ (angles sum to 180°).



Since the angle of incidence equals the angle of reflection, we have $\angle A_3 A_2 O = \angle A_1 A_2 O = \alpha = \angle A_2 A_3 O$ by equal radii. So, $\angle A_3OA_2 = \angle A_2 OA_1 = 180° - 2\alpha$. Thus, the angles at the centre are always equal. The cue ball returns to A_1 iff

$k(180° - 2\alpha) = 360°c \Rightarrow k(90° - \alpha) = 180°c$,

for some positive integers c and k. Since α must be real, $90° - \alpha$ must be either a rational or irrational number.

Case I: If $90° - \alpha$ is rational, it can be expressed in the from $\frac{m}{n}$, where m and n are positive, relatively prime integers. Thus $k\cdot\frac{m}{n}=180°c$. We may take $k = 180°n$ and then $m = c$, so for all α rational, the cue ball will return to A_1.

Case II: If $90° - \alpha$ is irrational, let $x = 90° - \alpha$. Then $kx = 180° \Rightarrow x=\frac{180°c}{k}$.

But x is irrational, so we have a contradiction.

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CBSE warm-up!

Term-II

CLASS-XI

Series-2 **Conic Section | Introduction to Three Dimensional Geometry | Derivatives | Probability**

Time Allowed : 2 hours
Maximum Marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. Let (3, 4, –1) and (–1, 2, 3) be the end points of a diameter of a sphere. Find the radius of the sphere.

2. If the equation of the parabola is $x^2 = -8y$, then find the equation of the directix and length of latus rectum.

OR

Find the equation of the parabola with focus (0, – 3) and directrix $y = 3$.

3. Find the derivative of
$f(x) = 1 + x + x^2 + x^3 + ... + x^{50}$ at $x = 1$.

4. An experiment involves rolling a pair of dice. The following events are recorded.
P : The sum is greater than 9.
Q : 1 occurs on either die.
R : The sum is atleast 8 and a multiple of 3.
Show that the pair P, Q and Q, R of these events are mutually exclusive.

5. Differentiate $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ w.r.t. x.

6. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, then find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

SECTION - B

7. Find the equation of ellipse whose foci are (0, ±6) and length of minor axis is 16 units. Also find the coordinates of the points where the ellipse cuts y-axis and its latus rectum.

OR

The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$. Find the equation of the hyperbola if its eccentricity is 2.

8. Find the derivative of

(i) $(6x^3 + 9x)(5x + 10)$ **(ii)** $\frac{5x+4}{x-3}$.

9. Find the coordinates of the point which divides the join of $P(2, -1, 4)$ and $Q(4, 3, 2)$ in the ratio 2 : 3 internally.

OR

Three consecutive vertices of a parallelogram $ABCD$ are $A(6, -2, 4)$, $B(2, 4, -8)$ and $C(-2, 2, 4)$. Find the coordinates of the fourth vertex.

10. Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive events.

SECTION - C

11. Find the point in yz-plane which is equidistant from the points $A(3, 2, -1)$, $B(1, -1, 0)$ and $C(2, 1, 2)$.

12. Differentiate $\cot\sqrt{x}$ w.r.t. x using first principle method.

OR

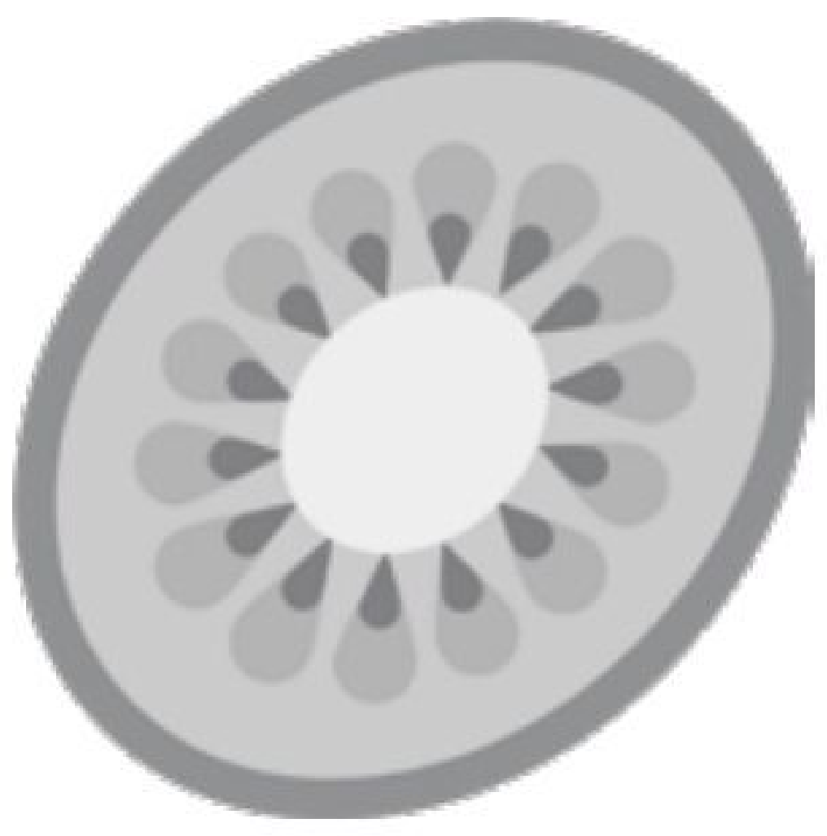
Find the derivative of $\sin x + \cos x$ using first principle of differentiation.

13. If E and F are two events such that $P(E)=\frac{1}{4}$, $P(F)=\frac{1}{2}$ and P (E and F) $=\frac{1}{8}$, find

(i) P(E but not F) (ii) P(F but not E)

CASE-BASED/DATA-BASED

14. In Maths period, Pankaj studied about the ellipse. On the same day, when he was helping his little brother in his study, he saw the image of kiwi fruit, cut in elliptical shape in picture book.



Consider the equation represented by fruit image as $\frac{x^2}{49}+\frac{y^2}{36}=1$ and answer the following questions.

(i) Find the vertices and sum of focal radii of the given curve.

(ii) Find the eccentricity and equation of directrices of the given curve.

SOLUTIONS

1. $\because$ $P(3, 4, -1)$ and $Q(-1, 2, 3)$ be the end points of the diameter of a sphere.

$\therefore$ Length of diameter $= PQ$

$$=\sqrt{(-1-3)^2+(2-4)^2+(3+1)^2}$$

$$=\sqrt{16+4+16}=\sqrt{36}=6 \text{ units}$$

$\therefore$ Radius $=\frac{6}{2}=3$ units

2. Comparing the given equation with standard form $x^2 = -4ay$, we get $a = 2$.

Therefore, the equation of directrix is $y = 2$ and the length of the latus rectum is $4a$, *i.e.*, 8.

OR

Since focus of the parabola is $(0, -3)$ and directrix $y = 3$.

$\therefore$ Equation of parabola will be $x^2 = -4ay$.

Here $a = 3$.

$\therefore$ Equation becomes $x^2 = -12y$.

3. We have, $f(x) = 1 + x + x^2 + x^3 + ... + x^{50}$

$\therefore$ $f'(x) = 0 + 1 + 2x + 3x^2 + ... + 50x^{49}$

So, $f'(1) = 1 + 2 + 3 + ... + 50 = \frac{50(50+1)}{2}$

$= 25 \times 51 = 1275$

4. Clearly, $P = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$.

$Q = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)\}$.

and $R = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$.

Here, $P \cap Q = \phi$ and $Q \cap R = \phi$

$\therefore$ Both P, Q and Q, R are mutually exclusive.

5. We have, $\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

$$=\frac{d}{dx}\left(x+\frac{1}{x}+2\right)=\frac{d}{dx}(x)+\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{d}{dx}(2)$$

$$=1+\left(\frac{-1}{x^2}\right)+0=1-\frac{1}{x^2}$$

6. Given, $P(A)=\frac{3}{5}, P(B)=\frac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B)$ $=\frac{3}{5}+\frac{1}{5}$

[$\because$ A and B are mutually exclusive events]

$\therefore$ $P(A \text{ or } B) = P(A \cup B)$ $=\frac{4}{5}$

7. Since foci of ellipse is $(0, \pm 6)$

$\therefore$ Major axis lies along y-axis.

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(d)
6.	(b)	**7.**	(a,d)	**8.**	(a,b,c,d)	**9.**	(a,d)	**10.**	(b,c)
11.	(b,c,d)	**12.**	(a,d)	**13.**	(c)	**14.**	(c)	**15.**	(a)
16.	(d)	**17.**	(9)	**18.**	(2)	**19.**	(4)	**20.**	(15)

Let equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ b>a$ ...(i)

Also, length of minor axis = $2a = 16 \Rightarrow a = 8$

$\therefore\quad b^2 - a^2 = 36 \Rightarrow b^2 - 64 = 36 \Rightarrow b = 10$

Putting the values of a and b in (i), we get

$\frac{x^2}{64}+\frac{y^2}{100}=1$, which is the required equation of ellipse.

Points where ellipse cuts y-axis = vertex = $(0, \pm b)$ = $(0, \pm 10)$.

Latus rectum $=\frac{2a^2}{b}=\frac{2(64)}{10}=12.8$ units

OR

The equation of the ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$.

$\therefore\quad a^2 = 25$ and $b^2 = 9$

Eccentricity of ellipse $=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

So, the coordinates of foci are $(\pm 4, 0)$

Since the coordinates of foci of hyperbola coincide with the foci of the ellipse. So the coordinates of foci of hyperbola are $(\pm 4, 0)$.

Let equation of required hyperbola is $\frac{x^2}{a'^2}-\frac{y^2}{b'^2}=1$

Let e' be the eccentricity of required hyperbola.

then $a'e' = 4 \Rightarrow 2a' = 4 \Rightarrow a' = 2$

$\therefore\quad b'^2 = a'^2(e'^2 - 1) \Rightarrow b'^2 = 4(4-1) = 12$

Thus, equation of required hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$.

8. **(i)** We have, $\frac{d}{dx}\left[(6x^3+9x)(5x+10)\right]$

$=\frac{d}{dx}\{30x^4+60x^3+45x^2+90x\}$

$= 30 \times 4x^3 + 60 \times 3x^2 + 45 \times 2x + 90 \times 1$

$= 120x^3 + 180x^2 + 90x + 90 = 30(4x^3 + 6x^2 + 3x + 3)$

(ii) $\frac{d}{dx}\left(\frac{5x+4}{x-3}\right)$

$$=\frac{(x-3)\cdot\frac{d}{dx}(5x+4)-(5x+4)\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$=\frac{(x-3)(5)-(5x+4)\cdot 1}{(x-3)^2}=\frac{5x-15-5x-4}{(x-3)^2}=\frac{-19}{(x-3)^2}$$

9. Let $R(x, y, z)$ be the required point.

If R divides PQ internally in the ratio 2 : 3, then

$$x=\frac{2\times 4+3\times 2}{2+3},\ y=\frac{2\times 3+3(-1)}{2+3}\ \text{and}\ z=\frac{2\times 2+3\times 4}{2+3}$$

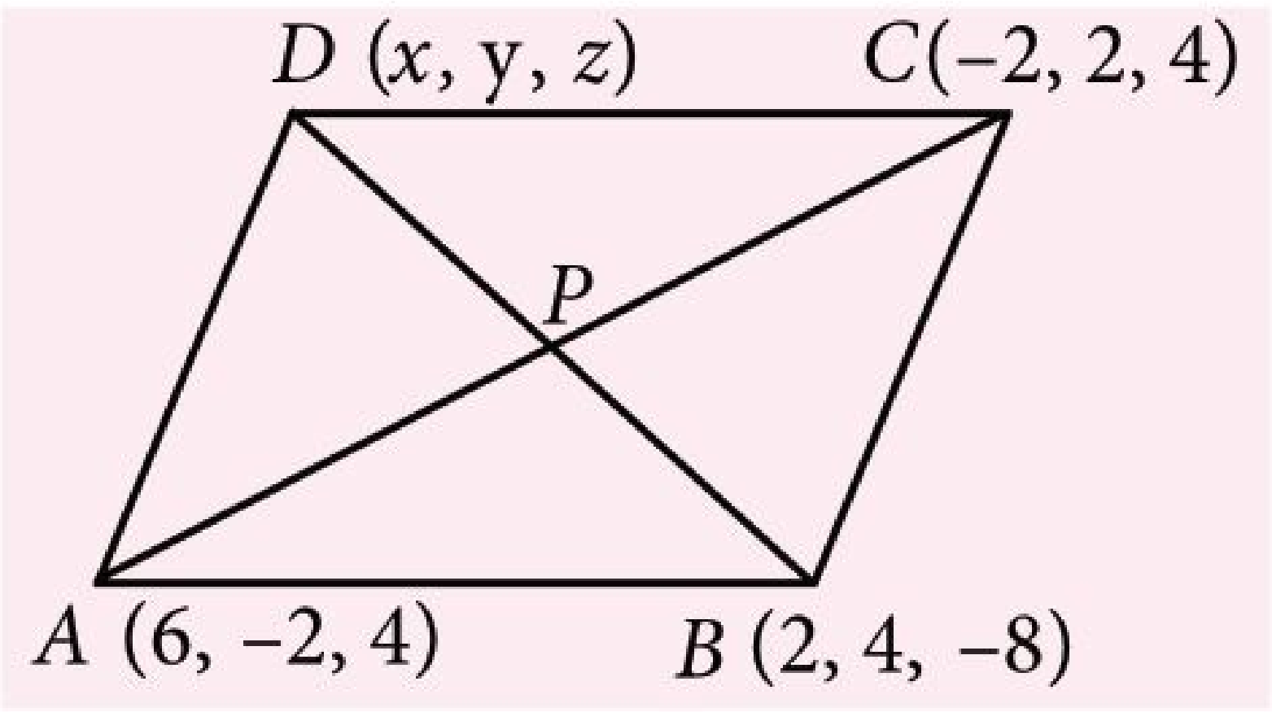
$$\left[\because\ (x,y,z)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)\right]$$

$\Rightarrow\ x=\frac{14}{5},\ y=\frac{3}{5}$ and $z=\frac{16}{5}$

So, the coordinates of point R are $\left(\frac{14}{5},\frac{3}{5},\frac{16}{5}\right)$.

OR

Let the coordinates of the fourth vertex be $D(x, y, z)$



Now by using the mid-point formula, mid-point of AC is $P\left(\frac{6-2}{2},\frac{-2+2}{2},\frac{4+4}{2}\right)$ *i.e.*, $P(2, 0, 4)$

Now, again using the mid-point formula for line BD, we get $P\left(\frac{x+2}{2},\frac{y+4}{2},\frac{z-8}{2}\right)$

Now, $2=\frac{x+2}{2}\Rightarrow x=2$; $0=\frac{y+4}{2}\Rightarrow y=-4$ and

$4=\frac{z-8}{2}\Rightarrow z=16$

Hence, the coordinates of fourth vertex D are $(2, -4, 16)$.

10. We have, $A = \{(1, 1)\}$, $B = \{(1, 2), (2, 1)\}$ and $C = \{(1, 3), (3, 1), (2, 2)\}$

(i) Since A consists of a single sample point, it is a simple event.

(ii) Since both B and C contain more than one sample point, hence each one of them is a compound event.

(iii) Since $A \cap B = \phi$,

$\therefore$ A and B are mutually exclusive events.

11. Let P be a point in the yz-plane. The point in the yz-plane is of the form $P(0, y, z)$.

$\because$ P is equidistant from $A(3, 2, -1)$, $B(1, -1, 0)$ and $C(2, 1, 2)$

$\therefore\quad AP = BP = CP \Leftrightarrow AP^2 = BP^2$ and $BP^2 = CP^2$

$\Rightarrow (0-3)^2 + (y-2)^2 + (z+1)^2$
$= (0-1)^2 + (y+1)^2 + (z-0)^2$

and $(0-1)^2 + (y+1)^2 + (z-0)^2$
$= (0-2)^2 + (y-1)^2 + (z-2)^2$

$\Rightarrow\quad 3y - z - 6 = 0$ ...(i)

and $4y + 4z - 7 = 0$ ...(ii)

Solving (i) and (ii), we get $y=\frac{31}{16}$ and $z=\frac{-3}{16}$

Hence, the required point is $\left(0, \frac{31}{16}, \frac{-3}{16}\right)$.

12. Let $f(x)=\cot\sqrt{x}$

From first principle, $\frac{d}{dx}f(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to0}\frac{\cot\sqrt{x+h}-\cot\sqrt{x}}{h}=\lim_{h\to0}\frac{\frac{\cos\sqrt{x+h}}{\sin\sqrt{x+h}}-\frac{\cos\sqrt{x}}{\sin\sqrt{x}}}{h}$$

$$=\lim_{h\to0}\frac{\sin\sqrt{x}\cos\sqrt{x+h}-\cos\sqrt{x}\sin\sqrt{x+h}}{h\sin\sqrt{x+h}\sin\sqrt{x}}$$

$$=\lim_{h\to0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{h\sin\sqrt{x+h}\cdot\sin\sqrt{x}}$$

$$=\lim_{h\to0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{(x+h-x)\sin\sqrt{x+h}\cdot\sin\sqrt{x}}$$

$$=\lim_{h\to0}\frac{-\sin(\sqrt{x+h}-\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})\sin\sqrt{x+h}\cdot\sin\sqrt{x}}$$

$$=\lim_{h\to0}\frac{\sin(\sqrt{x+h}-\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})}\ \frac{-1}{(\sqrt{x+h}+\sqrt{x})\sin\sqrt{x+h}\cdot\sin\sqrt{x}}$$

$$=\frac{-1}{2\sqrt{x}\sin\sqrt{x}\sin\sqrt{x}}=\frac{-\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$$

OR

Let $f(x) = \sin x + \cos x$

By first principle, $f'(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$

$$f'(x)=\lim_{h\to0}\frac{\sin(x+h)+\cos(x+h)-\sin x-\cos x}{h}$$

$$=\lim_{h\to0}\frac{\sin x\cos h+\cos x\sin h+\cos x\cos h-\sin x\sin h-\sin x-\cos x}{h}$$

$$=\lim_{h\to0}\frac{\sin h(\cos x-\sin x)+\sin x(\cos h-1)+\cos x(\cos h-1)}{h}$$

$$=\lim_{h\to0}\frac{\sin h}{h}(\cos x-\sin x)+\lim_{h\to0}\frac{\sin x(\cos h-1)}{h}+\lim_{h\to0}\frac{\cos x(\cos h-1)}{h}$$

$= \cos x - \sin x + \sin x\,(0) + \cos x\,(0)$

$$\left[\because\ \lim_{h\to0}\frac{\sin h}{h}=1 \text{ and } \lim_{h\to0}\frac{\cos h-1}{h}=0\right]$$

$= \cos x - \sin x$

13. Given, $P(E)=\frac{1}{4}, P(F)=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{8}$

(i) $P(E \text{ but not } F) = P(E\cap\bar{F}) = P(E) - P(E\cap F)$

$$=\frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}.$$

(ii) $P(F \text{ but not } E) = P(F\cap\bar{E}) = P(F) - P(F\cap E)$

$= 1/2 - 1/8 = 4 - 1/8 = 3/8$

14. (i) Here, $a = 7$, $b = 6$

∴ Vertices are $(\pm a, 0)$ *i.e.*, $(\pm7, 0)$

Also, sum of focal radii = Length of major axis = $2a = 14$.

(ii) Eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{36}{49}}=\frac{\sqrt{13}}{7}$

Also, equation of directrices are given by $x=\pm\frac{a}{e}$

i.e., $x=\pm\frac{7}{\sqrt{13}/7}=\pm\frac{49}{\sqrt{13}}$

◆

SAMURAI SUDOKU

SOLUTION · JANUARY 2022

1	6	4	2	8	7	5	3	9				1	2	9	3	6	5	8	7	4
9	7	2	5	3	6	1	4	8				5	4	3	2	8	7	6	9	1
8	3	5	9	1	4	2	7	6				8	7	6	1	9	4	5	2	3
6	5	1	4	2	8	3	9	7				3	9	4	5	7	8	2	1	6
4	2	3	1	7	9	6	8	5				7	8	1	6	4	2	9	3	5
7	8	9	6	5	3	4	2	1				2	6	5	9	3	1	4	8	7
3	1	7	8	4	5	9	6	2	7	3	1	4	5	8	7	2	3	1	6	9
2	4	6	7	9	1	8	5	3	4	2	6	9	1	7	8	5	6	3	4	2
5	9	8	3	6	2	7	1	4	8	9	5	6	3	2	4	1	9	7	5	8
						4	7	5	6	8	2	1	9	3						
						3	8	1	5	7	9	2	4	6						
						6	2	9	3	1	4	7	8	5						
9	1	5	7	4	6	2	3	8	9	4	7	5	6	1	7	4	9	2	3	8
4	7	8	2	5	3	1	9	6	2	5	8	3	7	4	8	2	5	9	6	1
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1	3	2	6	8	4	9	7	5				2	4	7	6	1	8	3	5	9
7	6	9	5	2	1	3	8	4				1	3	6	5	9	7	8	4	2
8	5	4	3	9	7	6	2	1				9	8	5	4	3	2	6	1	7
5	4	1	8	3	2	7	6	9				6	9	2	1	5	4	7	8	3
3	8	6	1	7	9	4	5	2				4	5	8	2	7	3	1	9	6
2	9	7	4	6	5	8	1	3				7	1	3	9	8	6	4	2	5

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Straight Lines

Time Taken : 60 Min.

Only One Option Correct Type

1. A variable straight line passes through a fixed point (a, b) intersecting the coordinate axes at A and B. If O is origin, then the locus of the centroid of ΔOAB is always lies on

(a) $bx + ay - 2xy = 0$ (b) $bx + ay - 3xy = 0$
(c) $ax + by - 3xy = 0$ (d) None of these

2. Points A and B are in the first quadrant. Point O is the origin. If the slope of OA is 1, slope of OB is 7 and $OA = OB$, then slope of AB is

(a) $-\frac{1}{5}$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{2}$

3. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

(a) 1 (b) 2
(c) 4 (d) None of these

4. If two of the lines represented by $x^4 + x^3y + cx^2y^2 - xy^3 + y^4 = 0$ bisect the angle between the other two, then the value of c is

(a) 0 (b) -1
(c) 1 (d) -6

5. If the lines $2(\sin a + \sin b)x - 2\sin(a - b)y = 3$ and $2(\cos a + \cos b)x + 2\cos(a - b)y = 5$ are perpendicular, then $\sin 2a + \sin 2b$ is equal to

(a) $\sin(a - b) - 2\sin(a + b)$
(b) $\sin 2(a - b) - 2\sin(a + b)$
(c) $2\sin(a - b) - \sin(a + b)$
(d) $\sin 2(a - b) - \sin(a + b)$

6. If pairs of lines $3x^2 - 2pxy - 3y^2 = 0$ and $5x^2 - 2qxy - 5y^2 = 0$ are such that each pair bisects the angle between the other pair, then pq is equal to

(a) -1 (b) -3 (c) -5 (d) -15

One or More Than One Option(s) Correct Type

7. If A and B are two fixed points whose coordinates are (3, 2) and (5, 4) respectively, then coordinates of point P if ABP is an equilateral triangle, can be

(a) $\left(4-\sqrt{3},\ 3+\sqrt{3}\right)$ (b) $\left(4+\sqrt{3},\ 3-\sqrt{3}\right)$
(c) $\left(3-\sqrt{3},\ 4+\sqrt{3}\right)$ (d) $\left(3+\sqrt{3},\ 4-\sqrt{3}\right)$

8. If the equation $12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$ represents a pair of perpendicular straight lines, then

(a) $p = 12$ (b) $p = -12$ (c) $q = 1$ (d) $q = -1$

9. For all values of θ, all the lines represented by the equation $(2\cos\theta + 3\sin\theta)x + (3\cos\theta - 5\sin\theta)y - (5\cos\theta - 2\sin\theta) = 0$

(a) passes through the point (1, 0)
(b) passes through the point (1, 1)
(c) passes through a fixed point whose reflection in the line $x + y = \sqrt{2}$ is $\left(\sqrt{2}-1, \sqrt{2}-1\right)$
(d) passes through the origin.

10. If p is the length of the perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$ and a^2, p^2, b^2 are in A.P. then ab is equal to

(a) p^2 (b) $\sqrt{2}p^2$ (c) $-\sqrt{2}p^2$ (d) $2p^2$

11. If the pair of lines represented by the equation $6x^2 + 17xy + 12y^2 + 22x + 31y + 20 = 0$ be $2x + 3y + p = 0$ and $3x + 4y + q = 0$, then

(a) $p + q = 9$ (b) $p^2 + q^2 = 41$
(c) $3p + 2q = 22$ (d) $4p + 3q = 31$

12. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then

(a) the lines will pass through a fixed point
(b) there will be a set of parallel lines
(c) all the lines intersect the line $x = x_1$
(d) all the lines will be parallel to the line $y = x_1$.

Comprehension Type

Paragraph for Q. No. 13 to 15

The base of an isosceles triangle is 4, the base angle is equal to 45°. A straight line cut the extension of the base at a point M at an angle θ and bisects the lateral side of the triangle which is nearer to M. Let A = the area of quadrilateral which the straight line cuts off with the given triangle.

13. The area A of the quadrilateral which the straight line cuts off from given triangle is

(a) $\frac{3+5\tan\theta}{1+\tan\theta}$ (b) $\frac{3+2\tan\theta}{1+\tan\theta}$

(c) $\frac{3+\tan\theta}{1+\tan\theta}$ (d) None of these

14. The possible range of values of A for different values of θ, is

(a) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (b) $(4, 5)$

(c) $\left(4, \frac{9}{2}\right)$ (d) $(3, 4)$

15. The length of portion of straight line inside the triangle may lie in the range

(a) $\left(\sqrt{2}, 2\right)$ (b) $(2, 4)$

(c) $\left(\frac{3}{2}, 2\right)$ (d) $\left(1, \sqrt{2}\right)$

Matrix Match Type

16. Match the following :

Column-I		Column-II	
P.	If the lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then the possible value of 2 λ can be	1.	2
Q.	If the points $(\lambda + 1, 1)$, $(2\lambda + 1, 3)$ and $(2\lambda + 2, 2\lambda)$ are collinear, then the possible value of λ can be	2.	4
R.	If line $y - x - 1 + \lambda = 0$ is equally inclined to axes and equidistant from the points $(1, -2)$ and $(3, 4)$, then λ is equal to	3.	-1

	P	Q	R
(a)	1	2	3
(b)	1	2	2
(c)	2	1	3
(d)	2	3	1

Numerical Answer Type

17. Among the lines, passing through $C(3, 1)$, BA is farthest from the origin and cuts the x-axis and y-axis at A and B respectively. Then $\frac{BC}{CA}$ is equal to ______.

18. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ________.

19. The coordinates of the feet of the perpendiculars from the vertices of a triangle on the opposite sides are (20, 25), (8, 16) and (8, 9). If the orthocentre of the triangle is (h, k), then $k^3 + h$ is equal to ________.

20. If $9x^2 + 2hxy + 4y^2 + 6x + 2fy - 3 = 0$ represents two parallel straight lines, then $|h + f|$ is equal to ________.

⁂

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

Term-II

CLASS-XII

Series 2 Vector | Three Dimensional Geometry | Probability

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. If A and B are two independent events, then find the probability of occurrence of atleast one of event A and B.

2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

3. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

4. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$.

5. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

6. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, then find the probability that at least one of them solves it.

OR

If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then find the value of $P(B'|A)$.

SECTION - B

7. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

8. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.

OR

If the sum of two unit vectors is a unit vector, then prove that the magnitude of their difference is $\sqrt{3}$.

9. A couple has 2 children. Find the probability that both are boys, if it is known that

(i) one of them is a boy.

(ii) the older child is a boy.

OR

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

10. By computing the shortest distance between the following pair of lines, determine whether they intersect or not?

$$\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}-\hat{k});\ \vec{r}=2\hat{i}-\hat{j}+\mu(\hat{i}-\hat{j}-\hat{k})$$

SECTION - C

11. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line, $\vec{r}=-\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also, find the image of P in this line.

12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

OR

If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

13. Find the perpendicular distance of the point $(1, 0, 0)$ from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

CASE-BASED/DATA-BASED

14. In a wedding ceremony, consists of father, mother, daughter and son line up at random for a family photograph, as shown in figure.

Based on the above information, answer the following questions.

(i) Find the probability that mother is at right end, given that son and daughter are together.

(ii) Find the probability that father and son are standing together, given that mother and daughter are standing together.

SOLUTIONS

1. P(atleast one of A and B)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ [$\because$ A, B are independent]

$= P(A) + P(B)\,[1 - P(A)] = 1 - P(A') + P(B)\,P(A')$

$= 1 - P(A')\,[1 - P(B)] = 1 - P(A')\,P(B')$

2. Given that, $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=5$

and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

On squaring both sides, we get

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\Rightarrow\ 3^2+4^2+5^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\Rightarrow\ 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-(9+16+25)$$

$$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{50}{2}\ \Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-25$$

3. Here, $x_1 = -3, y_1 = 1, z_1 = 5; a_1 = -3, b_1 = 1, c_1 = 5;$

$x_2 = -1, y_2 = 2, z_2 = 5$ and $a_2 = -1, b_2 = 2, c_2 = 5$

$$\text{Now, }\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

$= 2(5 - 10) - 1(-15 + 5) + 0 = -10 + 10 = 0$

Therefore, lines are coplanar.

4. Let $\vec{a}=2\hat{i}-3\hat{k}$ and $\vec{b}=4\hat{j}+2\hat{k}$

The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a}\times\vec{b}|$.

$$\text{Now, }\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -3 \\ 0 & 4 & 2 \end{vmatrix}=12\hat{i}-4\hat{j}+8\hat{k}$$

$$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{(12)^2+(-4)^2+(8)^2}=\sqrt{144+16+64}$$

$=\sqrt{224}=4\sqrt{14}$ sq. units.

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (b) **2.** (d) **3.** (b) **4.** (d) **5.** (b)
6. (d) **7.** (a,b) **8.** (a,c) **9.** (b,c) **10.** (b,c)
11. (a,b,c,d) **12.** (b,c) **13.** (a) **14.** (d)
15. (a) **16.** (d) **17.** (9) **18.** (5)
19. (3385) **20.** (8)

5. The given lines are

$$l_1 : \frac{x-0}{1} = \frac{y-0}{-1} = \frac{z-0}{\frac{1}{k}}$$

$$l_2 : \frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$$

$\because$ l_1 is perpendicular to l_2

$$\therefore \quad 1(1)+(-1)\left(\frac{1}{2}\right)+\left(\frac{1}{k}\right)(-1)=0$$

$$\Rightarrow \quad 1-\frac{1}{2}-\frac{1}{k}=0 \;\Rightarrow\; \frac{1}{2}=\frac{1}{k} \Rightarrow k=2$$

6. Let A, B and C be respectively the events of solving problem by three students and $P(A)$, $P(B)$ and $P(C)$ be their probability of solving the problem respectively.

$\therefore$ $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$

Required probability $= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$

($\because$ A, B and C are independent $\therefore$ $\bar{A}$, $\bar{B}$ and $\bar{C}$ are also independent)

$= 1 - [1 - P(A)]\,[1 - P(B)]\,[1 - P(C)]$

$$=1-\frac{2}{3}\times\frac{3}{4}\times\frac{5}{6} =1-\frac{5}{12}=\frac{7}{12}$$

OR

Given, A and B are independent events.

Also, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Now, $P(B'|A) = \dfrac{P(B' \cap A)}{P(A)}$

$= \dfrac{P(B')P(A)}{P(A)}$ [$\because$ A, B are independent events]

$= P(B') = 1 - P(B) = 1-\frac{1}{4}=\frac{3}{4}$

7. E : 'a total of 8' and F : 'red die resulted in a number less than 4'

i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$

i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

Hence, $E \cap F = \{(5, 3), (6, 2)\}$,

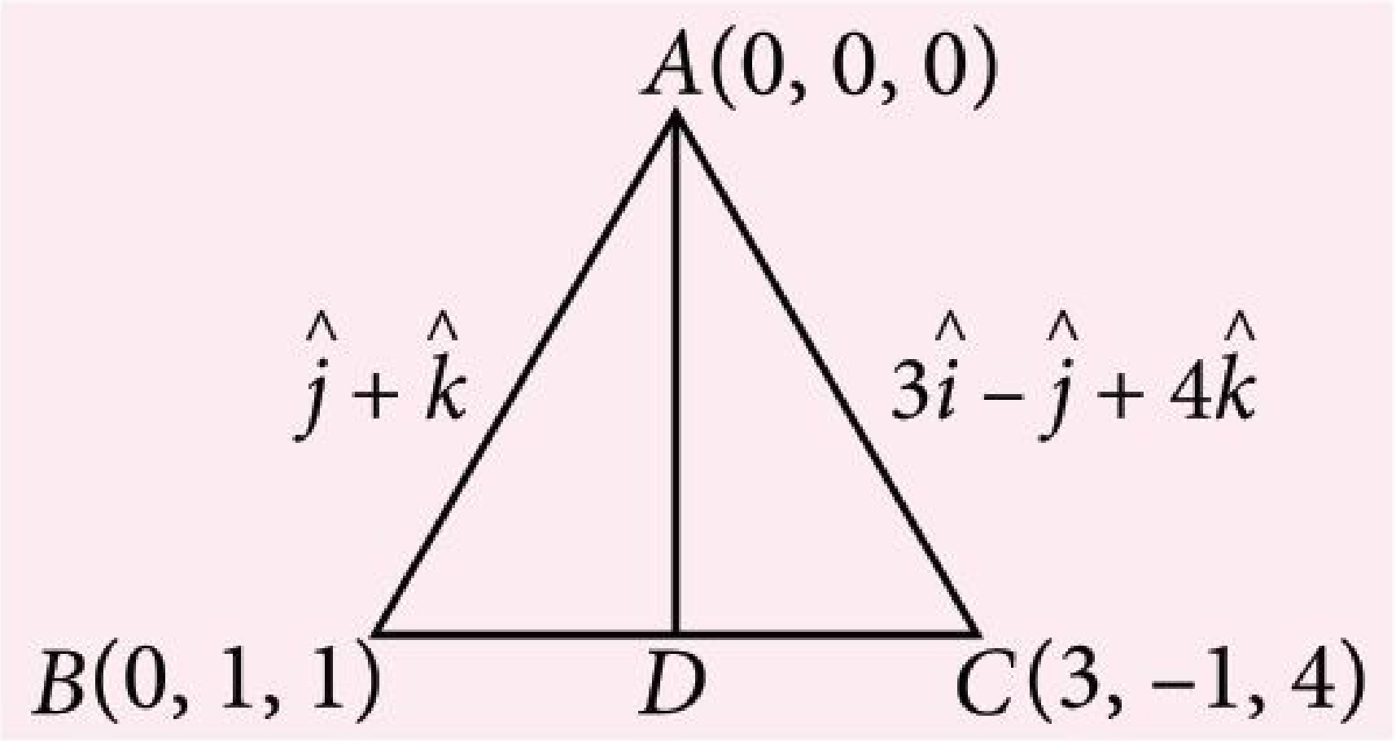
$P(E) = 5/36$, $P(F) = 18/36$, $P(E \cap F) = 2/36$

$\therefore$ Required probability $= P(E|F)$

$$=\frac{P(E\cap F)}{P(F)}=\frac{2/36}{18/36}=\frac{2}{18}=\frac{1}{9}$$

8. Take point A as origin $(0, 0, 0)$.

$\therefore$ Coordinates of B are $(0, 1, 1)$ and coordinates of C are $(3, -1, 4)$.



Let D be the mid point of BC and AD is the median of ΔABC.

$$\therefore \quad \text{Coordinates of } D \equiv \left(\frac{3+0}{2}, \frac{-1+1}{2}, \frac{4+1}{2}\right) \equiv \left(\frac{3}{2}, 0, \frac{5}{2}\right)$$

$$\text{So, length of } AD=\sqrt{\left(\frac{3}{2}-0\right)^2+(0)^2+\left(\frac{5}{2}-0\right)^2}$$

$$=\sqrt{\frac{9}{4}+\frac{25}{4}}=\frac{\sqrt{34}}{2} \text{ units}$$

OR

Given, $\hat{a}+\hat{b}=\hat{c}$

Now, $(\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})=\hat{c}\cdot\hat{c} \Rightarrow \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{b}+\hat{b}\cdot\hat{a}=\hat{c}\cdot\hat{c}$

$\Rightarrow 1+\hat{a}\cdot\hat{b}+1+\hat{a}\cdot\hat{b}=1 \Rightarrow 2\,\hat{a}\cdot\hat{b}=-1$ ...(i)

Now, $(\hat{a}-\hat{b})^2=(\hat{a}-\hat{b})\cdot(\hat{a}-\hat{b})$

$=\hat{a}\cdot\hat{a}-\hat{a}\cdot\hat{b}-\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b} =1-\hat{a}\cdot\hat{b}-\hat{a}\cdot\hat{b}+1$

$=2-2\hat{a}\cdot\hat{b}=2-(-1)$ [Using (i)]

$= 3$

$\therefore |\hat{a}-\hat{b}|=\sqrt{3}$

9. Let $B_i(i = 1, 2)$ and $G_i(i = 1, 2)$ denote the i^{th} child is a boy or a girl respectively.

Then sample space is,

$S = \{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$

Let A be the event that both are boys, B be the event that one of them is a boy and C be the event that the older child is a boy.

$A = \{B_1B_2\}$, $B = \{G_1B_2, B_1G_2, B_1B_2\}$

$C = \{B_1B_2, B_1G_2\} \Rightarrow A \cap B = \{B_1B_2\}$ and $A \cap C = \{B_1B_2\}$

(i) Required probability $= P(A|B)$

$$=\frac{P(A\cap B)}{P(B)}=\frac{1/4}{3/4}=\frac{1}{3}$$

(ii) Required probability $= P(A|C)$

$$=\frac{P(A\cap C)}{P(C)}=\frac{1/4}{2/4}=\frac{1}{2}$$

OR

We have, $S = \{1, 2, 3, 4, 5, 6\}$ and A be the event that number is even $= \{2, 4, 6\}$

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$$\Rightarrow \quad P(A)=\frac{3}{6}=\frac{1}{2}$$

B be the event that number is red = {1, 2, 3}

$$\Rightarrow \quad P(B)=\frac{3}{6}=\frac{1}{2} \text{ and } A\cap B=\{2\}$$

$$\Rightarrow \quad P(A\cap B)=\frac{1}{6} \qquad \text{...(i)}$$

$$\text{Also, } P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4} \qquad \text{...(ii)}$$

From (i) and (ii), $P(A)\cdot P(B)\neq P(A\cap B)$

So, A and B are not independent.

10. The equations of the lines are

$\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}-\hat{k})$, $\vec{r}=2\hat{i}-\hat{j}+\mu(\hat{i}-\hat{j}-\hat{k})$

Here, $\vec{a}_1=\hat{i}-\hat{j}, \vec{b}_1=2\hat{i}-\hat{k}$ and

$\vec{a}_2=2\hat{i}-\hat{j}, \vec{b}_2=\hat{i}-\hat{j}-\hat{k}$.

The S.D. between the given lines $=\left|\dfrac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

Now, $\vec{a}_2-\vec{a}_1=\hat{i}$

$$\Rightarrow \quad \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & -1\\ 1 & -1 & -1\end{vmatrix}=-\hat{i}+\hat{j}-2\hat{k}$$

$$\Rightarrow \quad |\vec{b}_1\times\vec{b}_2|=\sqrt{(-1)^2+1^2+(-2)^2}=\sqrt{6}$$

$$\therefore \quad \text{S.D.}=\left|\frac{(-\hat{i}+\hat{j}-2\hat{k})\cdot\hat{i}}{\sqrt{6}}\right|=\frac{1}{\sqrt{6}}(\neq 0)$$

⇒ The given lines do not intersect.

11. The given line is

$\vec{r}=-\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$

Its cartesian equation is

$$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda(\text{say}) \qquad \text{...(i)}$$

Any point Q on (i) is $(2\lambda-1, 3\lambda+3, -\lambda+1)$

Also, the given point is $P(5, 4, 2)$.

Now d.r's of the line PQ are

$(2\lambda-1-5, 3\lambda+3-4, -\lambda+1-2)$

$=(2\lambda-6, 3\lambda-1, -\lambda-1)$.

For PQ to be ⊥ to (i), we must have

$(2\lambda-6)\cdot 2+(3\lambda-1)\cdot 3+(-\lambda-1)\cdot(-1)=0$

$\Rightarrow \quad 14\lambda-14=0 \Rightarrow \lambda=1$

$\therefore \quad Q$ is (1, 6, 0)

which is the foot of ⊥ from P on line (i).

$$\text{Now, } PQ=\sqrt{(5-1)^2+(4-6)^2+(2-0)^2}$$

$$=\sqrt{24}=2\sqrt{6} \text{ units.}$$

Further if $R(\alpha, \beta, \gamma)$ is the image of P in line (i), then

$$\frac{\alpha+5}{2}=1, \frac{\beta+4}{2}=6, \frac{\gamma+2}{2}=0$$

$\Rightarrow \quad \alpha=-3, \beta=8, \gamma=-2$

$\therefore \quad$ Image of P in line (i) is $R(-3, 8, -2)$.

12. $|\vec{a}|=|\vec{b}|=|\vec{c}|$ (Given) ...(i)

and $\vec{a}\cdot\vec{b}=0, \vec{b}\cdot\vec{c}=0, \vec{c}\cdot\vec{a}=0$ ...(ii)

Let $(\vec{a}+\vec{b}+\vec{c})$ be inclined to vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ by angles α, β and γ respectively. Then

$$\cos\alpha=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{a}|^2+0+0}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} \qquad \text{[Using (ii)]}$$

$$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{...(iii)}$$

$$\text{Similarly, } \cos\beta=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{...(iv)}$$

$$\text{and } \cos\gamma=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|} \qquad \text{...(v)}$$

From (i), (iii), (iv) and (v), we get

$\cos\alpha=\cos\beta=\cos\gamma \Rightarrow \alpha=\beta=\gamma$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to the vector $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Also the angle between them is given as

$$\alpha=\cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right), \quad \beta=\cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right),$$

$$\gamma=\cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

OR

Two non-zero vectors are parallel if and only if their cross product is zero vector.

So, we have to prove that cross product of $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ is a zero vector.

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$(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{a}\times\vec{b})-(\vec{a}\times\vec{c})-(\vec{d}\times\vec{b})+(\vec{d}\times\vec{c})$

Since, it is given that $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$

Also, $\vec{d}\times\vec{b}=-\vec{b}\times\vec{d}$, $\vec{d}\times\vec{c}=-\vec{c}\times\vec{d}$

Therefore,

$(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{c}\times\vec{d})-(\vec{b}\times\vec{d})+(\vec{b}\times\vec{d})-(\vec{c}\times\vec{d})=\vec{0}$

Hence, $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where

$\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

13. Any point on the given line,

$$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=k\text{ (say)} \qquad \text{...(i)}$$

is $R(2k+1, -3k-1, 8k-10)$

If this is the foot of the $\perp$ from $P(1, 0, 0)$ on (i), then

$(2k+1-1)\cdot 2+(-3k-1-0)\cdot(-3)+(8k-10-0)\cdot 8=0$

$\Rightarrow\quad 4k+9k+3+64k-80=0 \Rightarrow 77k=77 \Rightarrow k=1.$

$\therefore\quad R$ is $(3, -4, -2)$.

This is the required foot of perpendicular.

Also, perpendicular distance $= PR$

$=\sqrt{(3-1)^2+(-4-0)^2+(-2-0)^2}=\sqrt{24}=2\sqrt{6}$ units.

Also equation of PR is $\dfrac{x-1}{2}=\dfrac{y}{-4}=\dfrac{z}{-2}$

14. Sample space is given by

{*MFSD*, *MFDS*, *MSFD*, *MSDF*, *MDFS*, *MDSF*,
FMSD, *FMDS*, *FSMD*, *FSDM*, *FDMS*, *FDSM*,
SFMD, *SFDM*, *SMFD*, *SMDF*, *SDMF*, *SDFM*
DFMS, *DFSM*, *DMSF*, *DMFS*, *DSMF*, *DSFM*},

where F, M, D and S represent father, mother, daughter and son respectively.

$\therefore\quad n(S)=24$

(i) Let A denotes the event that mother is at right end.

$\therefore\quad n(A)=6$

and B denotes the event that son and daughter are together.

$\therefore\quad n(B)=12$

Also, $n(A\cap B)=4$

$$\therefore\quad P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}=\frac{4/24}{12/24}=\frac{1}{3}$$

(ii) Let A denotes the event that father and son are standing together.

$\therefore\quad n(A)=12$

and B denotes the event that mother and daughter are standing together.

$\therefore\quad n(B)=12$

Also, $n(A\cap B)=8$

$$\therefore\quad P(A\,|\,B)=\frac{P(A\cap B)}{P(B)}=\frac{8/24}{12/24}=\frac{2}{3}$$

◆ ◆

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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 8 : Differential Equations

Time Taken : 60 Min.

Only One Option Correct Type

1. The degree of the differential equation satisfying $\sqrt{1+x^2}+\sqrt{1+y^2}=A\left(x\sqrt{1+y^2}+y\sqrt{1+x^2}\right)$ is

(a) 1 (b) 2 (c) 3 (d) 4

2. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{xy[x^2\sin y^2+1]}$ is

(a) $x^2(\cos y^2-\sin y^2-2ce^{-y^2})=2$

(b) $y^2(\cos x^2-\sin y^2-2ce^{-y^2})=2$

(c) $x^2(\cos y^2-\sin y^2-e^{-y^2})=4$

(d) $y^2(\sin y^2-\cos x^2-2ce^{-y^2})=4$

3. The equation of the curve which is such that portion of the axis of x cut off between the origin and the tangent at any point is proportional to the ordinate of that point is

(a) $x=y(C-K\log y)$ (b) $\log x=Ky^2+C$

(c) $x^2=y(C-K\log y)$ (d) None of these

(where C is an arbitrary constant and K is constant of proportionality)

4. Consider the differential equation $\frac{dy}{dx}+y\tan x=x\tan x+1$. Then

(a) the integral curves satisfying the differential equation and given by $y=x+c\sin x$.

(b) the angle at which the integral curves cut the y-axis is $\pi/2$.

(c) tangents to all the integral curves at their point of intersection with y-axis are parallel.

(d) none of these

5. If for the differential equation $y'=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$, the general solution is $y=\frac{x}{\log|Cx|}$, then $\phi(x/y)$ is given by

(a) $-x^2/y^2$ (b) y^2/x^2 (c) x^2/y^2 (d) $-y^2/x^2$

6. The point at which the normal to the curve $y=\ln x$ is perpendicular to the chord joining the points, (1, 0) and (e, 1), is

(a) $[e-1, \ln(e+1)]$ (b) $[e-1, \ln(e-1)]$

(c) $[e+1, \ln(e+1)]$ (d) None of these

One or More than One Option(s) Correct Type

7. The solution of $\frac{xdx+ydy}{xdy-ydx}=\sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$ is

(a) $\sqrt{x^2+y^2}=a\sin(\tan^{-1}y/x)+C$

(b) $\sqrt{x^2+y^2}=a\cos(\tan^{-1}y/x)+C$

(c) $\sqrt{x^2+y^2}=\tan(\sin^{-1}y/x)+C$

(d) $y=x\tan\left(C+\sin^{-1}\frac{1}{a}\sqrt{x^2+y^2}\right)$

8. Which of the following pair(s) is/are orthogonal?

(a) $16x^2+y^2=c$ and $y^{16}=kx$

(b) $y=x+ce^{-x}$ and $x+2=y+ke^{-y}$

(c) $y=cx^2$ and $x^2+2y^2=k$

(d) $x^2-y^2=c$ and $xy=k$

(where c and k are arbitrary constant)

9. The solutions of $(x+y+1)\,dy=dx$ are

(a) $x+y+2=Ce^y$

(b) $x+y+4=C\log y$

(c) $\log(x+y+2)=Cy$

(d) $\log(x+y+2)=C+y$

10. The solution of $\left(\frac{dy}{dx}\right)^2+2y\cot x\frac{dy}{dx}=y^2$ is

(a) $y-\frac{c}{1+\cos x}=0$ (b) $y=\frac{c}{1-\cos x}$

(c) $x=2\sin^{-1}\sqrt{c/2y}$ (d) None of these

11. The normal at a general point (a, b) on curve makes an angle θ with x-axis which satisfies $b(a^2\tan\theta - \cot\theta) = a(b^2 - 1)$. The equation of curve can be

(a) $y = e^{x^2/2} + c$ (b) $\log(ky^2) = x^2$
(c) $y = ke^{x^2/2}$ (d) $x^2 + y^2 = k$

12. Solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0$ are given by

(a) $y + e^{-x} = C$ (b) $y - e^{-x} = C$
(c) $y + e^x = C$ (d) $y - e^x = C$

Comprehension Type

Paragraph for Q. No. 13 to 15

Consider the differential equation

$$xyp^2 - (x^2 - y^2)p - xy = 0, \text{ where } p = \frac{dy}{dx}.$$

13. A solution of the above differential equation passing through (2 , 2) is

(a) $x^2 + y^2 = 8$ (b) $y^2 = 2x$
(c) $xy = 4$ (d) $x^2y = 1$

14. Another solution of above differential equation satisfying the point(2, 3) is

(a) $xy = 5$ (b) $x^2 + y^2 = 13$
(c) $y^2 - x^2 = 5$ (d) $y^2 = 4x + 1$

15. The equations of orthogonal trajectories of the solutions of the given differential equation are

(a) $xy = c_1, x^2 - y^2 = c_2$
(b) $xy = c_1, x^2 + y^2 = c_2$
(c) $x^2 - y^2 = c_1, x^2 + y^2 = c_2$
(d) None of these

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	Solution of $x\frac{dy}{dx} - y = (x-1)e^x$ is given by	1.	$-\frac{1}{xy} + \log y = c$ (where c is an arbitrary) constant)
Q.	Solution of the differential equation $ydx + (x + x^2y)\,dy$ is given by	2.	$y = e^x + cx$ (where c is an arbitrary constant)
R.	Solution of the differential equation $(x + y + 2)dx + (2x + 2y - 1)dy = 0$ is given by	3.	$2(x + y + 2) + 5\ln(x + y - 3) = x + c$, (where c is an arbitrary constant)

	P	Q	R
(a)	1	2	3
(b)	2	3	2
(c)	2	3	1
(d)	2	1	3

Numerical Answer Type

17. If $y = C_1e^{2x} + C_2e^x + C_3e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then $a^2 + b^2 + c^2$ is equal to_________.

18. If $6 + f''(x) + f'(x) = x^2 + f^2(x)$ is the differential equation of a curve and let P be the point of minima of this curve, then the number of tangents which can be drawn from P to the circle $x^2 + y^2 = 4$ is _________.

19. If $y = x\sin(\ln x)$ is the solution of

$$x^2\left(\frac{dy}{dx}\right)^2 - (\lambda - 2)xy\left(\frac{dy}{dx}\right) + 2y^2 - x^2 = 0,$$ then the value of λ is _________.

20. If the differential equation corresponding to the family of curves, $y = (A + Bx)e^{3x}$ is given by $\frac{d^2y}{dx^2} = a\frac{dy}{dx} + by$, then $(a - b)$ equals _________.

◆◆

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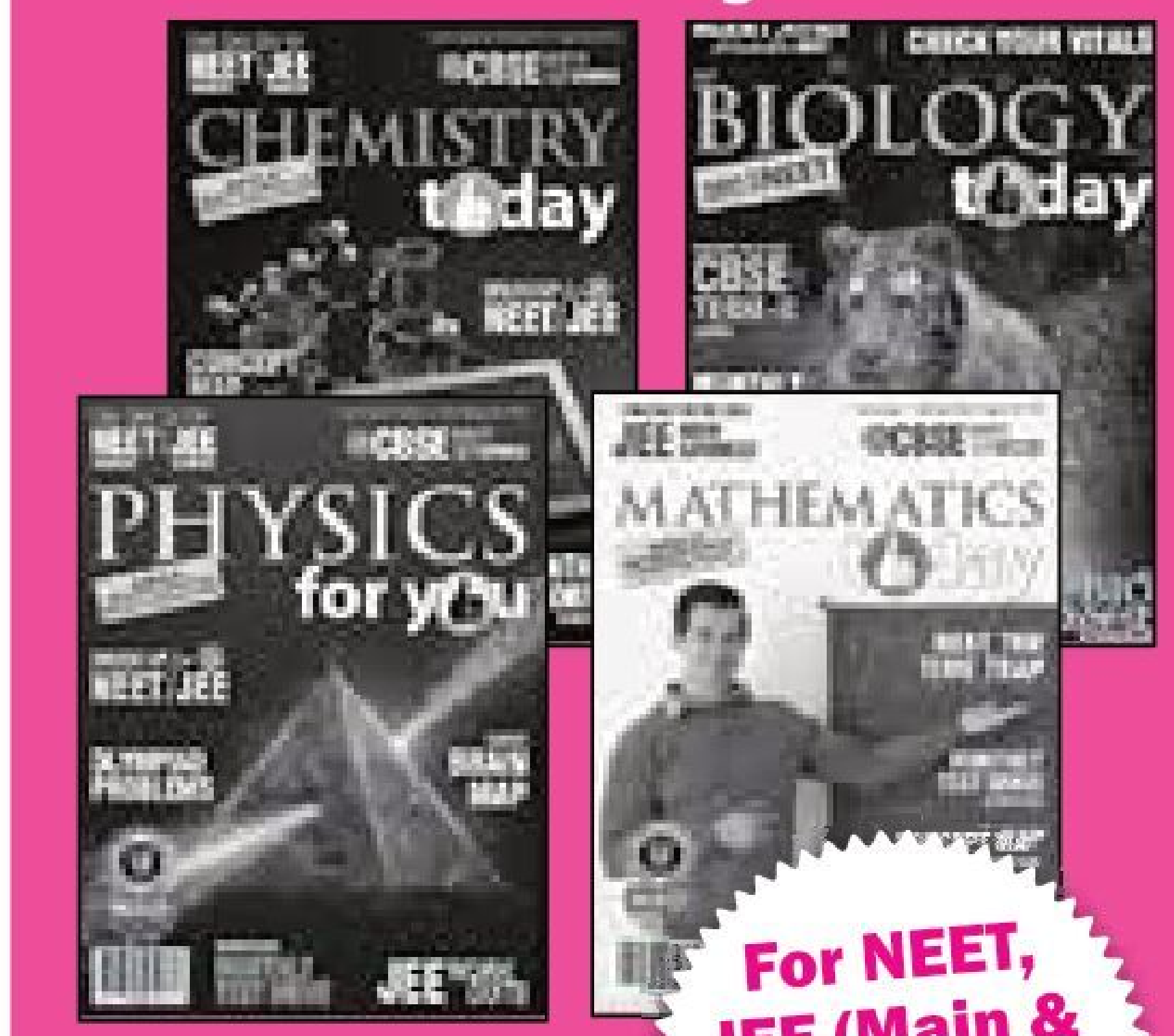

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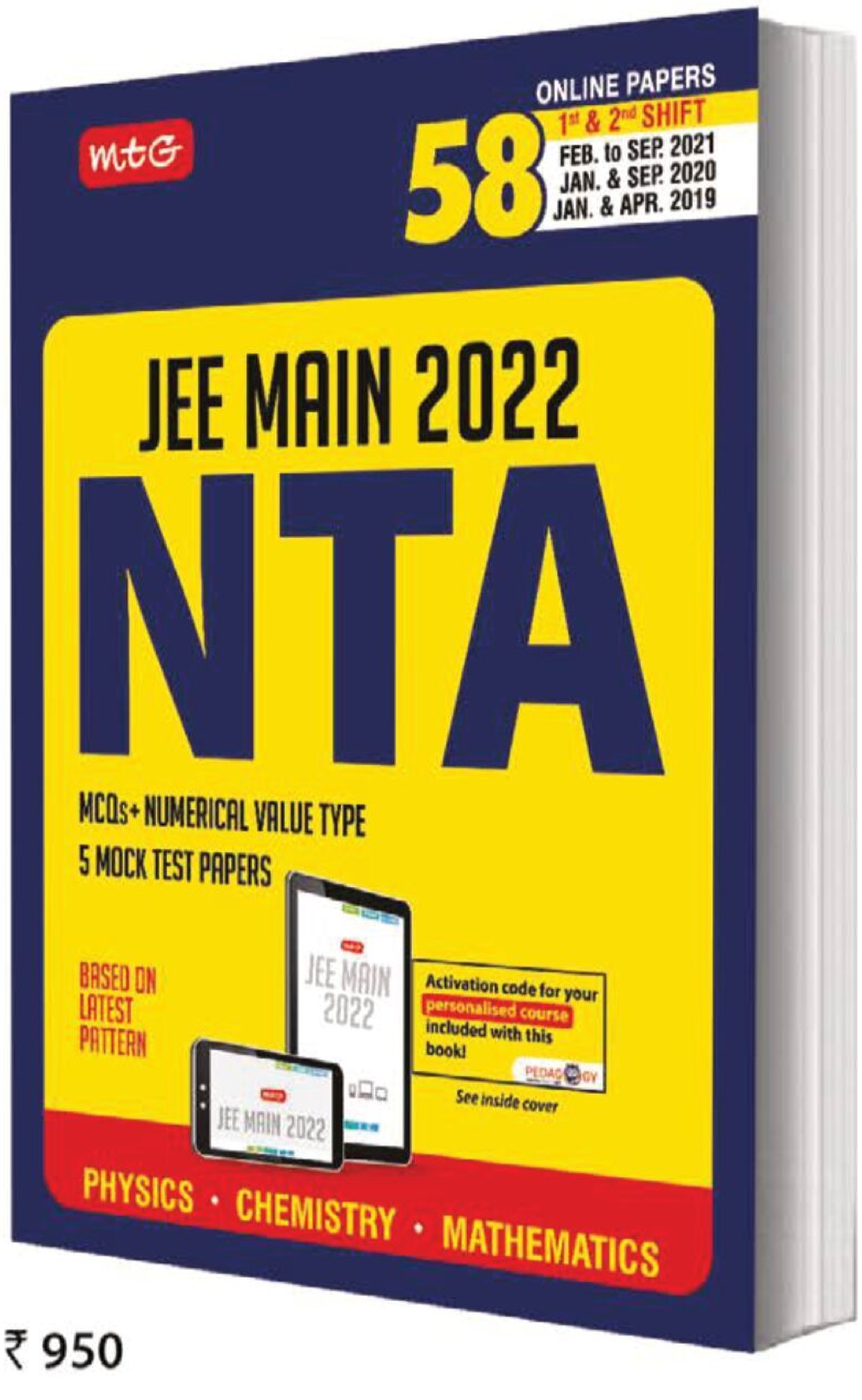
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